Extended LMP and Financial Transmission Rights

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Introduction

The unit commitment and economic dispatch problem introduces non-convexities into the electricity market model. Because of these non-convexities, there may be no set of energy prices that supports the economic dispatch as the market clearing solution. This gives rise to the uplift that balances the difference between optimal profits and actual profits at the given energy prices. Extended locational marginal prices (ELMP) derive from the convex hull of the unit commitment and economic dispatch problem. These ELMP prices minimize the uplift associated with the difference between the market clearing solution and the optimal commitment and dispatch. This minimum uplift includes transmission congestion and loss revenues that determine the revenue adequacy of financial transmission rights.

Electricity Market Model

The stylized version of the unit commitment and dispatch problem for a fixed demand $\mathbf{y}$ is formulated in (Gribik, Hogan, and Pope 2007) as:

$$v\left(\{\mathbf{y}_t\}\right) = \inf_{\mathrm{g,d,on,start}} \sum_t \sum_i \left(StartCost_{it} \cdot start_{it} + NoLoad_{it} \cdot on_{it} + GenCost_{it}\left(g_{it}\right)\right)$$

subject to

$$m_{it} \cdot on_{it} \le g_{it} \le M_{it} \cdot on_{it} \qquad \forall i,t$$

$$-ramp_{it} \le g_{it} - g_{i,t-1} \le ramp_{it} \qquad \forall i,t$$

$$start_{it} \le on_{it} \le start_{it} + on_{i,t-1} \qquad \forall i,t$$

$$start_{it} = 0 \text{ or } 1 \qquad \forall i,t$$

$$on_{it} = 0 \text{ or } 1 \qquad \forall i,t$$

$$\mathbf{e}^T\left(\mathbf{g}_t - \mathbf{d}_t\right) - LossFn_t\left(\mathbf{d}_t - \mathbf{g}_t\right) = 0 \qquad \forall t$$

$$Flow_{kt}\left(\mathbf{g}_t - \mathbf{d}_t\right) \le \overline{F}_{kt}^{\max} \qquad \forall k,t$$

$$\mathbf{d}_t = \mathbf{y}_t \qquad \forall t.$$

Indices:
- nodes *i* (and unit at node)
- time periods *t*
- transmission constraints *k*.

Variables:

$$start_{it} = \begin{cases} 0 \text{ if unit i is not started in period t} \\ 1 \text{ if unit i is started in period t} \end{cases}$$

$$on_{it} = \begin{cases} 0 \text{ if unit i is off in period t} \\ 1 \text{ if unit i is on in period t} \end{cases}$$

$g_{it} =$ output of unit i in period t

$\mathbf{d}_t =$ vector of nodal demands in period t.

Constants:

$\mathbf{y}_t =$ vector of nodal loads in period t

$m_{it} =$ minimum output from unit i in period t if unit is on

$M_{it} =$ maximum output from unit i in period t if unit is on

$ramp_{it} =$ maximum ramp from unit i between period t-1 and period t

$StartCost_{it} =$ Cost to start unit i in period t

$NoLoad_{it} =$ No load cost for unit i in period t if unit is on

$\overline{F}_{kt}^{\max} =$ Maximum flow on transmission constraint k in period t.

Functions:

$GenCost_{it}(\cdot) =$ Production cost above No Load Cost to produce energy from unit i in period t

$LossFn_t(\cdot) =$ Losses in period t as a function of net nodal withdrawals

$Flow_{kt}(\cdot) =$ Flow on constraint k in period t as a function of net nodal injections.

This simplified unit commitment and dispatch problem serves to illustrate the ideas. However, the basic results in (Gribik, Hogan, and Pope 2007) are more general and would encompass explicit consideration of demand bidding, minimum run times, integer decision variables for piecewise characterizations of non-convex functions and related non-convex formulations.

Let $\mathbf{start}^*, \mathbf{on}^*, \mathbf{g}^*, \mathbf{d}^*$ be a solution to the unit commitment and economic dispatch problem. Hence,

$$v(\{\mathbf{y}_t\}) = \sum_t \sum_i \left(StartCost_{it} \cdot start_{it}^* + NoLoad_{it} \cdot on_{it}^* + GenCost_{it}\left(g_{it}^*\right) \right).$$

A version of the convex hull formulation utilizes the dual with respect to the load, loss and transmission constraints, as in (Gribik, Hogan, and Pope 2007).

$$
\begin{aligned}
& v^h\left(0,\left\{\bar{F}_{kt}^{\max}\right\},\left\{\mathbf{y}_t\right\}\right) \equiv \\
& \sup_{p,\lambda,\mu}\left\{
\begin{array}{l}
+\sum_t \mathbf{p}_t^T \mathbf{y}_t \\
+\left\{
\begin{array}{ll}
\inf_{\text{g,d,on,start}} \left(
\begin{array}{l}
\sum_t \sum_i \left(StartCost_{it} \cdot start_{it} + NoLoad_{it} \cdot on_{it} + GenCost_{it}\left(g_{it}\right)\right) \\
-\sum_t \lambda_t \left(\mathbf{e}^T \left(\mathbf{g}_t - \mathbf{d}_t\right) - LossFn_t\left(\mathbf{d}_t - \mathbf{g}_t\right)\right) \\
+\sum_t \sum_k \mu_{kt} \left(Flow_{kt}\left(\mathbf{g}_t - \mathbf{d}_t\right) - \bar{F}_{kt}^{\max}\right) \; -\sum_t \mathbf{p}_t^T \mathbf{d}_t
\end{array}
\right) & \\
\text{subject to} & \\
m_{it} \cdot on_{it} \le g_{it} \le M_{it} \cdot on_{it} & \forall i,t \\
-ramp_{it} \le g_{it} - g_{i,t-1} \le ramp_{it} & \forall i,t \\
start_{it} \le on_{it} \le start_{it} + on_{i,t-1} & \forall i,t \\
start_{it} = 0 \text{ or } 1 & \forall i,t \\
on_{it} = 0 \text{ or } 1 & \forall i,t
\end{array}
\right\}
\end{array}
\right\} \qquad (1) \\
& \text{subject to} \\
& \mu_{kt} \ge 0 \qquad \forall t.
\end{aligned}
$$

Suppose we have a solution for the ELMP prices, $\mathbf{p},\boldsymbol{\lambda},\boldsymbol{\mu}$. Also assume that we have a solution, $\mathbf{start^o},\mathbf{on^o},\mathbf{g^o},\mathbf{d^o}$, for the interior unit commitment and dispatch problem in (1). This is a market clearing solution under the ELMP prices. This solution would be optimal if we simply announced the ELMP prices and let all the market participants solve for a profit maximizing solution. Note that this solution includes an endogenous choice of demand as well as the level of generation.

Given the solution to the commitment variables, $\mathbf{start^o},\mathbf{on^o}$, the inner problem reduces to a standard economic dispatch problem. With sufficient regularity conditions on the transmission and loss constraints, a strong duality theorem yields the result that the market clearing solution satisfies the constraints (Bazaraa, Sherali, & Shetty, 2006, Theorem 6.2.5). In particular, given the market clearing solution for the commitment variables, and holding these fixed, we assume that the remaining problem reduces to a standard economic dispatch problem with no duality gap.[1] Therefore we satisfy the saddle point conditions and:

$$\begin{aligned}
&\sum_t \lambda_t \left(\mathbf{e}^T \left(\mathbf{g}_t^o - \mathbf{d}_t^o \right) - LossFn_t \left(\mathbf{d}_t^o - \mathbf{g}_t^o \right) \right) = 0, \\
&\mathbf{e}^T \left(\mathbf{g}_t^o - \mathbf{d}_t^o \right) - LossFn_t \left(\mathbf{d}_t^o - \mathbf{g}_t^o \right) = 0, \\
&\sum_t \sum_k \mu_{kt} \left(Flow_{kt} \left(\mathbf{g}_t^o - \mathbf{d}_t^o \right) - \overline{F}_{kt}^{\max} \right) \ = 0, \\
&Flow_{kt} \left(\mathbf{g}_t^o - \mathbf{d}_t^o \right) \le \overline{F}_{kt}^{\max}.
\end{aligned}$$

It follows that the ELMP defined above as the solution $\mathbf{p}$ is also an ELMP solution for the corresponding problem including the loss and transmission limits included as constraints.

$$v^h\left(\{\mathbf{y}_t\}\right) \equiv \sup_p \left\{ \begin{array}{l} +\sum_t \mathbf{p}_t^T \mathbf{y}_t \\ + \left\{ \begin{array}{ll} \inf\limits_{\mathrm{g,d,on,start}} \left(\begin{array}{l} \sum_t \sum_i \left(StartCost_{it} \cdot start_{it} + NoLoad_{it} \cdot on_{it} + GenCost_{it}\left(g_{it}\right)\right) \\ -\sum_t \mathbf{p}_t^T \mathbf{d}_t \end{array} \right) & \\ \text{subject to} & \\ m_{it} \cdot on_{it} \le g_{it} \le M_{it} \cdot on_{it} & \forall i,t \\ -ramp_{it} \le g_{it} - g_{i,t-1} \le ramp_{it} & \forall i,t \\ start_{it} \le on_{it} \le start_{it} + on_{i,t-1} & \forall i,t \\ start_{it} = 0 \text{ or } 1 & \forall i,t \\ on_{it} = 0 \text{ or } 1 & \forall i,t \\ \mathbf{e}^T\left(\mathbf{g}_t - \mathbf{d}_t\right) - LossFn_t\left(\mathbf{d}_t - \mathbf{g}_t\right) = 0 & \forall t \\ Flow_{kt}\left(\mathbf{g}_t - \mathbf{d}_t\right) \le \overline{F}_{kt}^{\max} & \forall k,t \end{array} \right\} \end{array} \right\} \qquad (2)$$

[1] The transmission constraints would easily admit to a feasible solution with no constraints binding, for example at zero load. The only difficulty then would be the loss equation which is nonlinear. The interior solution point would be possible for a slightly perturbed version of the problem with a small violation, and these perturbed problems would satisfy the (strong) Slater condition. Similarly, a linearized loss equation would be sufficient to meet the regularity condition. (Bazaraa, Sherali, and Shetty 2006) Here we assume that some sufficient regularity condition would apply.

In addition, we have both original cost function and the corresponding convex hull values are the same.

$$v\left(\left\{\mathbf{y}_t\right\}\right)=v\left(0,\left\{\bar{F}_{kt}^{\max}\right\},\left\{\mathbf{y}_t\right\}\right),$$
$$v^h\left(\left\{\mathbf{y}_t\right\}\right)=v^h\left(0,\left\{\bar{F}_{kt}^{\max}\right\},\left\{\mathbf{y}_t\right\}\right).$$

When interpreting the nature of the ELMP, therefore, this equivalence of (1) and (2) allows us to choose the more convenient form in characterizing the prices, solutions, and the associated uplift or duality gap.

For example, the market clearing solution is an economic dispatch (and commitment) for demand $\mathbf{d}^{\mathbf{o}}$. We have

$$v\left(\left\{\mathbf{d}_t^0\right\}\right)\geq v^h\left(\left\{\mathbf{d}_t^0\right\}\right)=\sup_{\tilde{p}}\left\{+\sum_t \tilde{\mathbf{p}}_t^T\mathbf{d}_t^0+\inf_{\mathrm{d}}\left(v\left(\left\{\mathbf{d}_t\right\}\right)-\sum_t \tilde{\mathbf{p}}_t^T\mathbf{d}_t\right)\right\}$$
$$=\sum_t \mathbf{p}_t^T\mathbf{d}_t^0+\inf_{\mathrm{d}}\left(v\left(\left\{\mathbf{d}_t\right\}\right)-\sum_t \mathbf{p}_t^T\mathbf{d}_t\right)=v\left(\left\{\mathbf{d}_t^0\right\}\right).$$

At this level of demand, there is no duality gap. Hence, the market clearing solution and the ELMP prices have a natural interpretation as the economic dispatch (and commitment) and associated LMPs for this endogenous level of demand, which may differ from the actual level of demand in $\mathbf{d}^*$.

Similarly, the duality gap equals the minimum uplift (Gribik, Hogan, and Pope 2007), and this uplift is:

$$v\left(\left\{\mathbf{y}_t\right\}\right)-v^h\left(\left\{\mathbf{y}_t\right\}\right)=\sum_t\sum_i\left(StartCost_{it}\cdot start_{it}^*+NoLoad_{it}\cdot on_{it}^*+GenCost_{it}\left(g_{it}^*\right)\right)-\sum_t \mathbf{p}_t^T\mathbf{y}_t$$
$$-\sum_t\sum_i\left(StartCost_{it}\cdot start_{it}^o+NoLoad_{it}\cdot on_{it}^o+GenCost_{it}\left(g_{it}^o\right)\right)+\sum_t \mathbf{p}_t^T\mathbf{d}_t^o$$

We can rewrite this uplift as:

$$
\begin{aligned}
&v\left(\left\{\mathbf{y}_t\right\}\right)-v^h\left(\left\{\mathbf{y}_t\right\}\right)=\sum_t\sum_i\left(StartCost_{it}\cdot start_{it}^{*}+NoLoad_{it}\cdot on_{it}^{*}+GenCost_{it}\left(g_{it}^{*}\right)\right)-\sum_t\mathbf{p}_t^T\mathbf{d}_t^{*}\\
&-\sum_t\sum_i\left(StartCost_{it}\cdot start_{it}^{o}+NoLoad_{it}\cdot on_{it}^{o}+GenCost_{it}\left(g_{it}^{o}\right)\right)+\sum_t\mathbf{p}_t^T\mathbf{d}_t^{o}\\
&=\left(\sum_t\mathbf{p}_t^T\mathbf{g}_t^{o}-\sum_t\sum_i\left(StartCost_{it}\cdot start_{it}^{o}+NoLoad_{it}\cdot on_{it}^{o}+GenCost_{it}\left(g_{it}^{o}\right)\right)\right)\\
&-\left(\sum_t\mathbf{p}_t^T\mathbf{g}_t^{*}-\sum_t\sum_i\left(StartCost_{it}\cdot start_{it}^{*}+NoLoad_{it}\cdot on_{it}^{*}+GenCost_{it}\left(g_{it}^{*}\right)\right)\right)\\
&+\sum_t\mathbf{p}_t^T\left(\mathbf{d}_t^{o}-\mathbf{g}_t^{o}\right)-\sum_t\mathbf{p}_t^T\left(\mathbf{d}_t^{*}-\mathbf{g}_t^{*}\right).
\end{aligned}
$$

Using the ELMP prices, the first term is the generator profit under the associated market clearing solution; the second term is the generator profit with the optimal unit commitment and dispatch solution; the third term is the congestion and loss surplus for the market clearing solution; and the fourth term is the congestion and loss surplus for the optimal unit commitment and dispatch.

In summary, using the ELMP prices, the minimum uplift decomposes into the difference between the market clearing generator profits and the economic dispatch generator profits, and the difference between the market clearing transmission revenues and the economic dispatch transmission revenues.

Financial Transmission Rights

The definition of financial transmission rights includes many variants that are obligations or options, account for losses or only for transmission congestion (Hogan 2002). In a lossless model, congestion rights admit a series of revenue adequacy conditions for obligations and options. In models with losses, unbalanced FTRs with losses included have a similar series of revenue adequacy conditions. Mixing the definitions, most notably by using congestion-only rights in a framework with losses, may violate some of these results. However, at present we are interested in an interpretation of the revenue adequacy results with ELMP prices, assume that we are dealing with losses and the FTRs include the losses.

The FTRs define a vector of net loads that are simultaneously feasible. Hence, we assume that:

$$
\begin{aligned}
&\mathbf{e}^T\left(\mathbf{FTR}_t\right)-LossFn_t\left(\mathbf{FTR}_t\right)=0,\\
&Flow_{kt}\left(\mathbf{FTR}_t\right)\le\bar{F}_{kt}^{\max}.
\end{aligned}
$$

Then with a market clearing solution, we have by the usual arguments that the FTRs are revenue adequate:

$$\sum_t \mathbf{p}_t^T \left(\mathbf{d}_t^o - \mathbf{g}_t^o\right) \geq \sum_t \mathbf{p}_t^T \left(\mathbf{FTR}_t\right).$$

In other words, the transmission congestion and loss surplus revenues under the market clearing solution would be greater than the payments for the FTRs. The FTRs would be revenue adequate with the market clearing solution implied by the ELMP prices. The difficulty is that revenue adequacy may not apply to the economic dispatch solution, $\mathbf{start}^*, \mathbf{on}^*, \mathbf{g}^*, \mathbf{d}^*$. This is another aspect of the difficulty that the economic dispatch may not be supported by any prices, including the ELMP prices $\mathbf{p}$.[2] The deficiency is bounded by the difference between the transmission revenues in the third and fourth terms of the decomposition above. In other words,

$$\sum_t \mathbf{p}_t^T \left(\mathbf{d}_t^o - \mathbf{g}_t^o\right) - \sum_t \mathbf{p}_t^T \left(\mathbf{d}_{it}^* - \mathbf{g}_t^*\right) \geq \sum_t \mathbf{p}_t^T \left(\mathbf{FTR}_t\right) - \sum_t \mathbf{p}_t^T \left(\mathbf{d}_{it}^* - \mathbf{g}_t^*\right).$$

The transmission revenue that would be collected under the market clearing solution would be sufficient to meet the obligations under the FTRs. However, this may not be true for the revenues under the economic dispatch, which is not a market clearing solution at the ELMP prices, even though the FTRs are simultaneously feasible. However, the uplift amount of $\sum_t \mathbf{p}_t^T \left(\mathbf{d}_t^o - \mathbf{g}_t^o\right) - \sum_t \mathbf{p}_t^T \left(\mathbf{d}_t^* - \mathbf{g}_t^*\right)$ is included in the decomposition of the total uplift that is minimized by the ELMP prices. This uplift payment would be enough to ensure revenue adequacy of FTRs under ELMP pricing.

References

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[2] Here we assume that we are dealing with the same grid. Revenue adequacy under ELMP prices is a different problem than assuring revenue adequacy when the capacity of the grid changes and the FTRs are not simultaneously feasible for any values of the integer variables.

Endnotes

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