# COORDINATING CONGESTION RELIEF ACROSS MULTIPLE REGIONS

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# COORDINATING CONGESTION RELIEF ACROSS MULTIPLE REGIONS

Michael D. Cadwalader, Scott M. Harvey, William W. Hogan and Susan L. Pope[1]

Market coordination of congestion relief in an electric grid with multiple regions implies trading across boundaries. Regional system operators provide coordination services within their respective regions, and exchange information with other system operators to secure coordinated congestion relief across regions. Reliability would be preserved through a system based on market bids. Nothing revolutionary would be required. Any two system operators could begin the process, later adding other regions to expand the scope of coordinated congestion relief.

## INTRODUCTION

An interconnected electric transmission grid inherently requires coordination of its use. In large networks, there may be multiple control areas with system operators responsible for different areas. Inevitably the multiple operators must have some procedure for exchanging information and making decisions that affect the patterns of use across the grid.

With the introduction of competition and greater regional trading, the North American Electric Reliability Council (NERC) assumed responsibility for developing new coordination mechanisms that include transmission loading relief (TLR) to curtail scheduled transactions to keep use of the grid within its secure capacity. The early approaches were not market oriented but relied upon a set of administrative priorities for congestion relief. It is widely recognized that some form of improved coordination is of

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great importance in preserving the reliability of the grid. Furthermore, it is widely recognized that coordination based on market principles would be of great importance in supporting a restructured electricity market.[2] Despite the need, the initial NERC trials in this direction have been disappointing and have not produced a workable approach. The market approach outlined here takes key elements of the initial TLR framework and builds on them to include price information and trading among regions.

This approach is a variant of a market-based procedure for coordination across multiple regions, adapting the earlier analysis in Cadwalader et al.[3] In particular, the present discussion selects one approach from among the elements of a larger set of alternatives and elaborates on the implications within the context of coordinated congestion relief. The discussion emphasizes the case where the participating regions include Independent System Operators (ISOs) who are prepared to provide aggregated bid information for schedule adjustments across regions. The call for improved coordinated congestion relief among ISOs motivates the analysis, but the coordination mechanism could be viewed as a process for coordination across any set of control areas and multiple system operators. The purpose here is to identify problem structures and information requirements for coordination across multiple regions.

## COORDINATION ACROSS REGIONS

System operators within regions must of necessity maintain responsibility for coordinating the many bilateral schedules, spot market transactions, and load balancing adjustments to keep within the limits of the transmission system.[4] The result of the process within a region produces a set of loads and generation outputs at each location. The resulting net load at each location within the region then contributes to the flows on the transmission grid. In an interconnected grid with multiple regions, the net loads within a region normally contribute to transmission flows outside the region. This is a special feature of electric networks, and this external effect can be significant. It can and does result in regional schedules that, in the aggregate, overload the transmission system. If system reliability is to be maintained, this transmission congestion cannot be allowed. Inevitably, therefore, the regional system operators must have some way to change the schedules and other transactions, in order to change the net loads at the various locations, to find a form of congestion relief that brings the resulting aggregate transmission flows within the capability of the system. The existing coordination mechanisms are poorly suited for a market environment.

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2 Congestion Management Working Group of the NERC Market Interface Committee, "Comparison of System Redispatch Methods for Congestion Management," September 1999.

3 Michael Cadwalader, Scott Harvey, William Hogan, and Susan Pope, "Market Coordination of Transmission Loading Relief Across Multiple Regions," Center for Business and Government, Harvard University, December 1, 1998.

4 William Hogan, "Competitive Electricity Market Design: A Wholesale Primer," Center for Business and Government, Harvard University, December 18, 1998.

The coordination process across regions should provide the foundation for market-based congestion relief. The individual regions would in the first instance determine a set of schedules for use of the transmission grid. These schedules may or may not include estimates of the expected schedules from other regions. Once in place, the schedules might not be consistent, such as when the combined schedules would violate some transmission constraint, and there would be a need for congestion relief. The proposed coordination process would then provide an adjustment process, operated through the regional control area system operators or security coordinators, that would redispatch the system to achieve a feasible solution that meets the standards of reliable, security-constrained, economic dispatch.

In outline, the process would consist of a series of steps:

**Market Scheduling and Balancing**

1. Each regional system operator receives bilateral schedules, spot market bids, and schedule adjustment bids.

2. Based on these bids, and an assumption about loop flows from other systems, each regional system operator produces an initial market-based schedule that balances its market.

3. Each system operator reports to the other system operators the resulting aggregate schedule and net loads. If the several regional schedules and the implied aggregate dispatch and power flows are simultaneously feasible, no congestion redispatch would be required. If the combined schedules are not feasible, any regional system operator can invoke the coordinated redispatch protocol.

**Redispatch Protocol**

4. Each regional system operator reports to the other regions its net loads and the locational congestion costs arising from its own constraints that would apply to adjustments in the net loads at any location in the grid. In addition, each regional system operator identifies the adjustment bids that would apply in its region for the net load adjustments that might be arranged for other regions.

5. Based on the information in step 4, each regional system operator updates its estimates of the net loads in other regions. Each regional system operator then reformulates its regional economic dispatch problem to include the adjustment bids and the associated congestion costs from the other regions. Each regional system operator performs a redispatch with this new economic dispatch formulation, including possible redispatch at locations outside its region.

6. If there are significant changes in the dispatch for any region, we return to step 4 and provide updated estimates of schedules, bids and prices. If the redispatch step requires no new adjustments in schedules, the result provides a coordinated redispatch solution for the regions.

The design objective for the coordination method is to provide enough information and follow a protocol such that the process arrives quickly at an acceptable solution that uses market bids and respects the various transmission constraints throughout the region. The coordination method could be applied to achieve an efficient market solution even in the absence of transmission constraints, which is another subject. Here the focus is on congestion relief in the face of transmission constraints.

The price information exchanged among the regional system operators includes the congestion costs and the adjustment bids that could be used for coordination purposes. The resulting adjustment problem for each system operator appears in the form of a simple economic dispatch problem of a familiar type. The only difference from a unilateral economic dispatch approach is that each system operator has available an appropriate representation of the adjustments possible in other systems, and all the complexity of the other systems is reduced to the congestion information combined with the adjustment bids.

The coordination protocol envisions system operators communicating with each other during the iterative process, using the bid information acquired from market participants in the first scheduling phase to decide on the subsequent redispatch adjustments. To simplify the coordination process and reduce the opportunities for strategic bidding by market participants, the redispatch process does not include subsequent revision of the bids from market participants once the redispatch protocol is invoked. The system operator can update the adjustment bids provided to other regions, but it uses the market bids provided initially by the market participants in its own region. Hence, the complications of the iterative process are encapsulated in the coordination among regional system operators, which is essentially a solution procedure and not an iterative market.

The discussion here describes a method that could apply to day-ahead or near-real time markets. In principle, the same process outline would apply within each market, and the coordination function applied would be for every such dispatch. The setting is an economic framework for redispatch costs. The assumption is that important matters like the timing of submissions to the various regions, data formats, boundary definitions, fixed charges for entering or exiting a region, and so on have been addressed and made compatible with an efficient economic framework. These are important matters, but they are not addressed here.

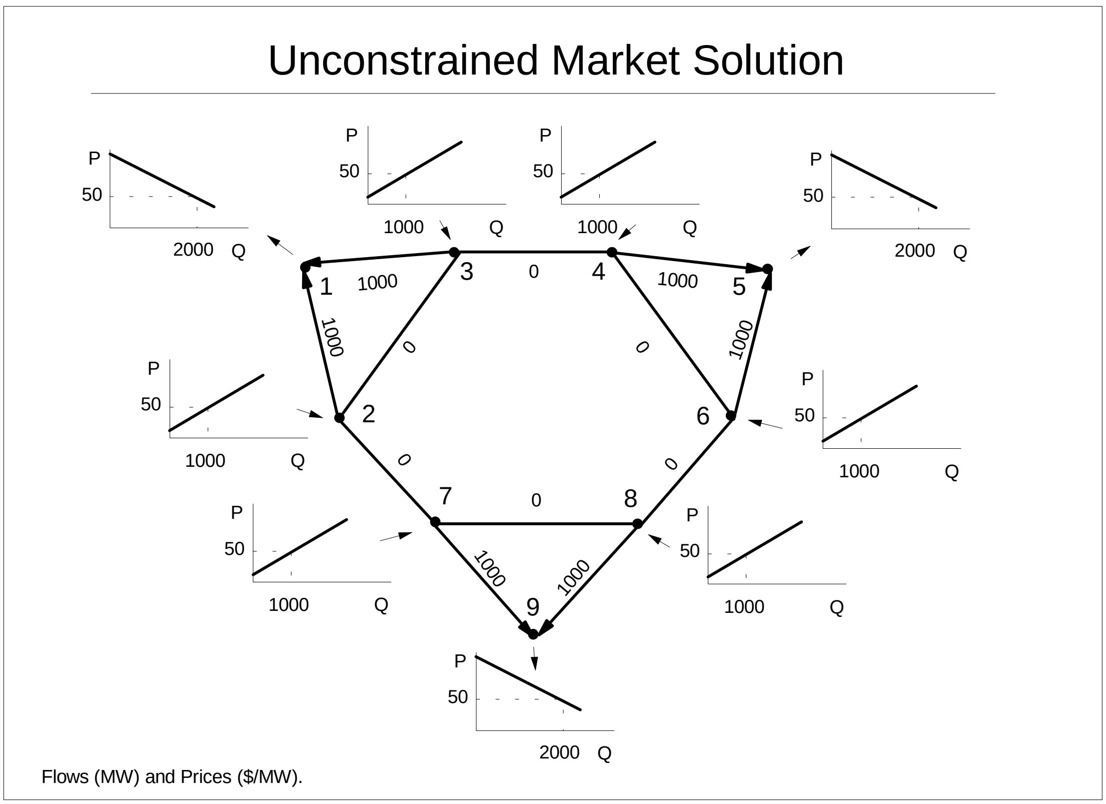
The details of the underlying model and coordination protocol appear in an appendix. An example application illustrates the analysis and information exchange. We follow this with a discussion of various issues regarding implementation and computational requirements.

## ILLUSTRATION OF THE REDISPATCH PROTOCOL

To illustrate the operation of the method outline above for a redispatch protocol, we summarize here a simple example on a DC-load network model without losses or contingency security constraints. Because of the nature of the particular coordination mechanism here, the addition of contingency constraints or losses should have no significant effect on the process, although it would complicate the illustration by requiring further details such as the constraints to be represented separately for each contingency.

Here a test problem with 9 buses and 12 lines utilizes the stylized network in the accompanying figure. Each line is assumed to have the same impedance characteristics. In the loss-less DC load formulation, this simplifies the verification of the flows, where the sum of the line flows around any loop must equal zero and the flows along any parallel paths are inversely related to the length of the path.

In addition, each bus in the network has a generator or a load. The loads appear at buses 1, 5 and 9 with the indicated demand curves. The other buses are shown as generator buses with identical supply curves.[5] (The intent of all this symmetry is to make the problem more transparent.) The demand curves could arise from bid in load, and the use of the linear demand could be replaced by step functions. Similarly, the generation supply curves could arise as the aggregation of many smaller bids from individual generators. In each case, the form of the supply and demand functions should not be too important as long as higher quantities of net supply are associated with higher bid price.

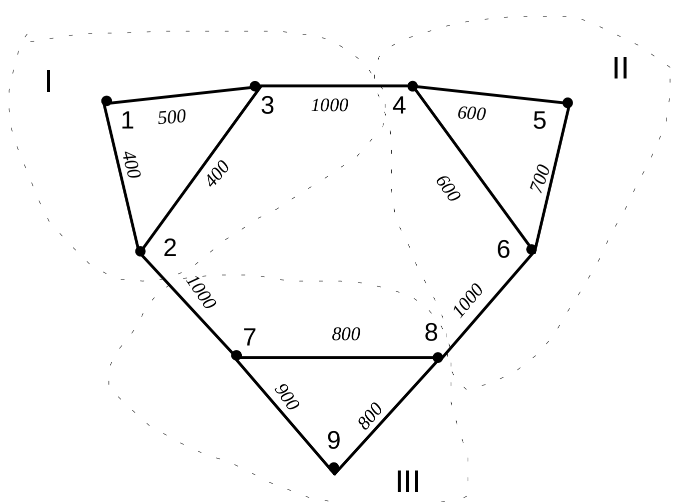



[5] To reduce the clutter in the figure, only one point is shown for each curve. The intercepts are at $110 and $20 for the demand and supply curves, respectively. The slopes all have an absolute value of 0.03.

We could suppose that the initial market schedules ignore transmission constraints. An initial equilibrium that balances the market would be an otherwise unconstrained solution in the network. We need not be too specific about the combination of arbitrage and market scheduling that produces this initial equilibrium. However, we take the simple starting case of equilibrium without transmission constraints. The figure shows the unconstrained solution, with a common system wide market price of $50 per MW.

To introduce the regional decomposition, we convert the example by identifying three different regions with the associated assignment of lines and buses. Each of the groupings in regions I, II, and III has its own system operator with responsibility for transmission constraints in its region and redispatch to achieve coordinated congestion relief, as indicated in the accompanying figure. Hence, the system operator in Region I is responsible for monitoring and managing the limits on flows within region I among locations 1, 2, and 3 as well as the regional interconnector on the line between locations 3 and 4. And so on for regions II and III.

## Regional Groupings and Transmission Limits

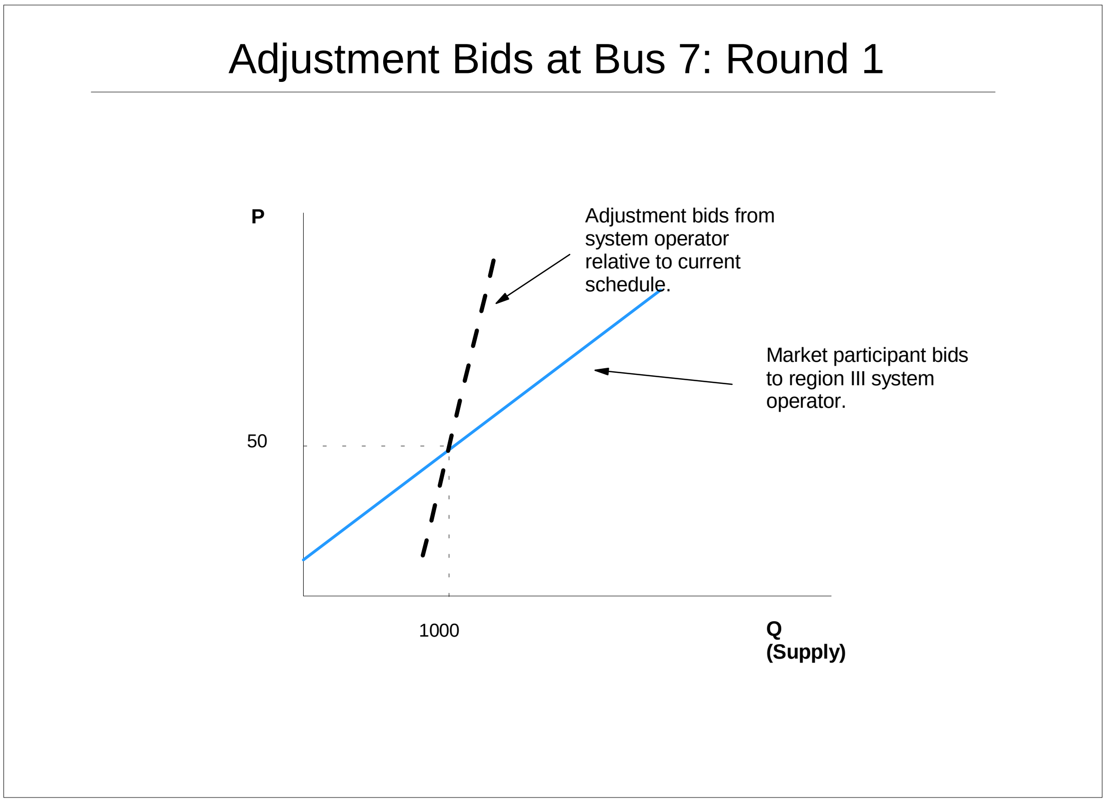



*Thermal limits on transmission lines (MW).*

In addition, the figure includes the description of the transmission constraints. For this illustration, all the constraints enter as thermal limits specifying the maximum allowable flow on each line. As can be seen by comparison with the previous figure, the unconstrained solution obtained from the initial market schedule would violate these transmission limits. This gives rise to the need for coordinated congestion relief.

To find a solution for the coordinated congestion relief problem, one or more of the regional system operators invokes the redispatch protocol. Once the redispatch protocol is invoked, each regional system operator provides three sets of information for use by the other regional operators: net loads from tentative schedules, congestion costs created by its constraints, and prices with adjustment bids for the locations in its region.

In the first round, the information is simple. The net loads for the tentative schedule for each region are just the net loads from the unconstrained market solution. The initial estimates of congestion costs are all zero. The only information of importance is the set of adjustments bids. For example, the adjustment bids at bus 7, provided by the system operator for region III, could appear as in the figure.




For the purposes of the example, we allow for the possibility that the detailed bids from the market participants would be complicated by many steps, intertemporal ramping limits, and so on. Hence, full construction and reporting of the exact participant bids across the full range of increments and decrements might require some significant effort. It turns out, however, that less is required for a successful coordinated congestion relief process. In effect, what is really needed is not the bid relationship across the full range at each stage, but only the bid price results for the current schedules and a rough approximation of the increments and decrements that would be available for redispatch.

Hence, the adjustment bids could be set conservatively by the regional system operators and need not correspond exactly to the detail in the underlying bids by the market participants. For simplicity, we assume that the adjustment bids appear on the form of increments and decrements relative to the current price, with the increments at \$0.2 per MW. Therefore, according to the assumed adjustment bids provided by the system operator, going from 1000 MW to 1001 MW of input at location 7 would raise the implied adjustment price at location 7 from the unconstrained \$50 to \$50.20, even though the actual bids from the market participants' supply curve would increased only to \$50.03. We include this type of approximation in the example to emphasize the point that the adjustment information provided by the system operators can be less detailed and only an approximation of the more complicated actual bids of the market participants. In the end, the important piece of information is the matching of \$50 and the 1000 MW, not so much the exact size of the adjustment bids.

We could consider each system operator adjusting simultaneously, but it is easier for the illustration if we address them in sequence. Suppose, for this sake of simplicity, we assume that each region works in turn, and we begin with region I. Given the information obtained from the other regions, the region I system operator formulates an economic dispatch problem that includes all the usual information for its region, augmented with the adjustment bids and congestion cost estimates supplied by the other regional system operators. Hence, the system operator for region I uses its full information about the market participant bids at locations 1, 2 and 3, and uses the approximate adjustments bids and congestion cost information to construct relative cost functions that apply to the other locations. In addition, the system operator in region I recognizes explicitly the constraints on the four transmission lines in its area of responsibility.

With the congestion cost and adjustments bids, the system operator in region I determines an efficient *system-wide* redispatch that would bring the power flows within the constraints for *its* region. Compared to the unconstrained solution, the system operator calls for adjustments of quantities and price to produce the new net inputs:

**Round 1**

| Location | | Tentative Schedule | Adjustment | New Schedule |
|---|---|---|---|---|
| 1 | p | 50.00 | 33.00 | 83.00 |
| | q | 2000.00 | -1100.00 | 900.00 |
| 2 | p | 50.00 | -16.22 | 33.78 |
| | q | -1000.00 | 540.51 | -459.49 |
| 3 | p | 50.00 | -6.54 | 43.46 |
| | q | -1000.00 | 218.11 | -781.89 |
| 4 | p | 50.00 | -1.32 | 48.68 |
| | q | -1000.00 | 43.87 | -956.13 |
| 5 | p | 50.00 | -1.43 | 48.57 |
| | q | 2000.00 | 47.60 | 2047.60 |
| 6 | p | 50.00 | -1.54 | 48.46 |
| | q | -1000.00 | 51.32 | -948.68 |
| 7 | p | 50.00 | -2.10 | 47.90 |
| | q | -1000.00 | 69.92 | -930.08 |
| 8 | p | 50.00 | -1.87 | 48.13 |
| | q | -1000.00 | 62.48 | -937.52 |
| 9 | p | 50.00 | -1.99 | 48.01 |
| | q | 2000.00 | 66.20 | 2066.20 |

This coordinated redispatch requires the largest schedule adjustments within region I, reflecting the greater impact on the local constraints. However, the redispatch does call for significant adjustments outside the region, with significant increases in load elsewhere to relieve the constraints in region I while maintaining an efficient market balance.

In addition to the changes in net loads throughout the system, the system operator in region I produces new estimates of the prices in its region and the congestion costs for its own constraints. Given the schedule adjustment, system operators from all the regions produce new prices to reflect the schedule net load adjustments. Note that the actual price changes reported by the other system operators reflect the quantity adjustments at the underlying response rate of the actual bids ($0.03 per MW) rather than the approximate estimates ($0.20 per MW) of the adjustments increments and decrements. This is one place in the procedure where the approximations connect to the underlying bids to assure movement towards a coordinated solution.

The congestion costs reported by region I are no longer zero. In the new schedule, the constraints on the lines connected to location 1 are both binding. Here we are treating location 1 as the reference bus, meaning that increase in load elsewhere would be balanced with an increase in generation at location 1. With this convention, the congestion cost at location 1 is always zero, and the congestion cost elsewhere is set relative to the reference price. This convention produces relative congestion prices that can be interpreted in this example as the marginal redispatch cost of supplying an additional MW from the respective bus to location 1, as in:

**Congestion Costs from Region I**

| **Bus** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** |
|---|---|---|---|---|---|---|---|---|---|
| | 0 | 49.22 | 39.54 | 41.78 | 42.52 | 43.26 | 46.98 | 45.50 | 46.24 |

Hence, increasing load at location 1 and balancing with generation at location 7 would increase congestion costs in region I by $46.98 per MW, and so on. Naturally, increasing load at location 7 would decrease the congestion costs by the same amount.

These congestion costs are provided by the system operator in region I who determines them in the usual way from the locational prices associated with its economic redispatch using its approximations of the adjustment bids for locations outside region I. The congestion costs can be obtained directly from the value of the binding constraints and the network shift factors, as shown in the appendix.[6]

With the new schedules, prices and congestion costs, the system operator in region II takes up its part of the task of finding a coordinated redispatch. For simplicity, the new adjustment bids from the other two system operators have the same form as above with the increments and decrements at $0.20 per MW set relative to the new prices. The system operator in region II now faces the problem of determining an economic redispatch with these proposed adjustment bids and the congestion costs estimates for the constraints in region I.

The combined result of the adjustment bids and congestion costs produces a new congestion-constrained supply curve at location 7, as seen from the perspective of the system operator in region II. As shown in the figure, the system operator in region II sees

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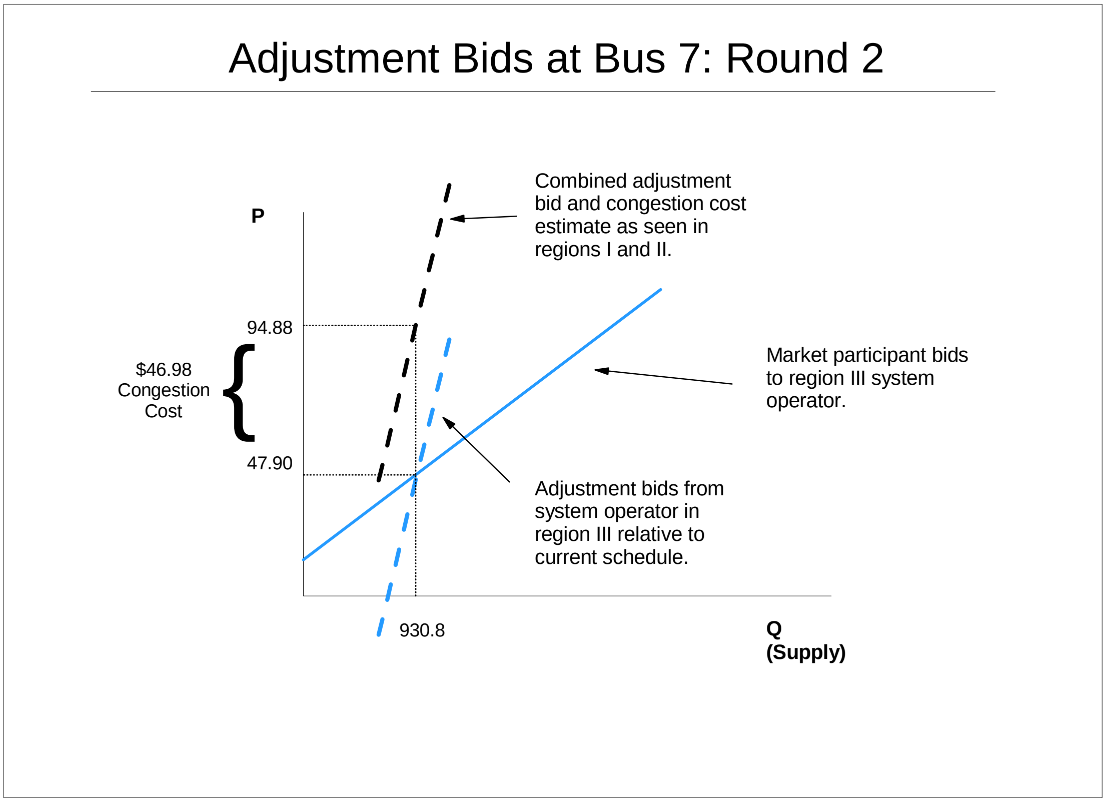
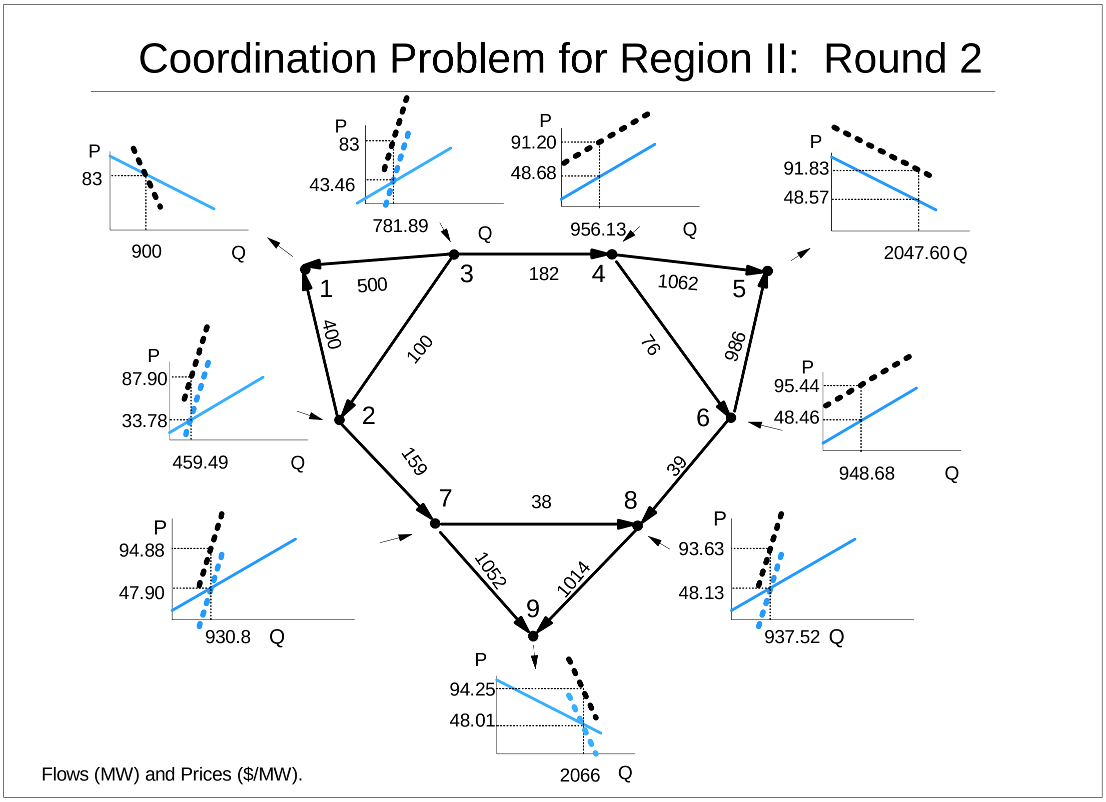
[6] The network shift factors and constraint multipliers for each iteration are shown in the appendix. Hence, for bus 2 we have $\$49.22=\lambda_I \nabla K_{I2}=$ 61.22*0.644+27.64*0.356. The congestion cost is the difference in the price at the reference bus and the price at the location as estimated by the region I system operator using its current available adjustment bids and external congestion costs. Therefore, in this round where the external congestion costs are assumed to be zero, the estimated congestion from region I is exactly the difference in prices at locations 1, 2, and 3. However, the estimated congestion costs are not yet consistent with the prices shown for the other locations where the system operators have reflected actual participants bids rather than the approximate adjustment bids. For example in the region I problem the price for the load at bus 9 is $36.76=$50-0.20(66.20) implying a congestion cost from region I of $46.24=$83-$36.76. However, the actual price at location 9 is $48.01=$50-0.03(66.20).

the apparent price as the sum of the current market price and the cost of congestion at location 7. Because the region I congestion costs are included in the adjusted incremental supply, the system operator in region II can treat the lines outside region II as unconstrained. The system operator needs to include explicit constraints only for its own region.




At the end of round 1, this same analysis applies to adjustment bids and congestion costs for all the locations in the grid. From the perspective of the system operator in region II, the current situation is as shown in the accompanying figure. The current schedules are feasible from the perspective of region I, because the system operator in region I has just completed its proposed set of schedule adjustments to move within the constraints in region I. But this set of net loads and line flows would not be acceptable for region II. In this case, for example, the power flows would not be feasible on the line from 4 to 5 or between 5 and 6. To arrange for congestion relief, the system operator takes the adjustments bids and congestion cost estimates to produce the problem as shown in the figure at the start of round 2. This initial set of schedules and flows is not a solution from the perspective of the system operator in region II. It must redispatch to obtain congestion relief.




The adjustment bids have been set to capture the effects of constraints outside of region II, and are shown by the heavy dashed lines in the figure. The system operator has these adjustment bids from external locations and the original market bids within its own region. In each case, the external adjustment bids and the internal market participant bids are adjusted up or down to reflect the cost of external congestion. The system operator knows the constraints that apply within region II, but it ignores the constraints outside region II. The system operator in region II uses this information to produce its own security constrained economic redispatch to meet the constraints in region II with a new set of schedule adjustments as in:

**Round 2**

| Location | | Tentative Schedule | Adjustment | New Schedule |
|---|---|---|---|---|
| 1 | p | 83.00 | -0.63 | 82.37 |
| | q | 900.00 | 20.91 | 920.91 |
| 2 | p | 33.78 | -0.41 | 33.38 |
| | q | -459.49 | 13.60 | -445.89 |
| 3 | p | 43.46 | -0.83 | 42.63 |
| | q | -781.89 | 27.56 | -754.33 |
| 4 | p | 48.68 | -17.18 | 31.50 |
| | q | -956.13 | 572.71 | -383.41 |
| 5 | p | 48.57 | 22.96 | 71.54 |
| | q | 2047.60 | -765.44 | 1282.16 |
| 6 | p | 48.46 | -0.18 | 48.28 |
| | q | -948.68 | 5.95 | -942.73 |
| 7 | p | 47.90 | -1.56 | 46.35 |
| | q | -930.08 | 51.85 | -878.24 |
| 8 | p | 48.13 | -0.94 | 47.19 |
| | q | -937.52 | 31.32 | -906.21 |
| 9 | p | 48.01 | -1.25 | 46.77 |
| | q | 2066.20 | 41.54 | 2107.73 |

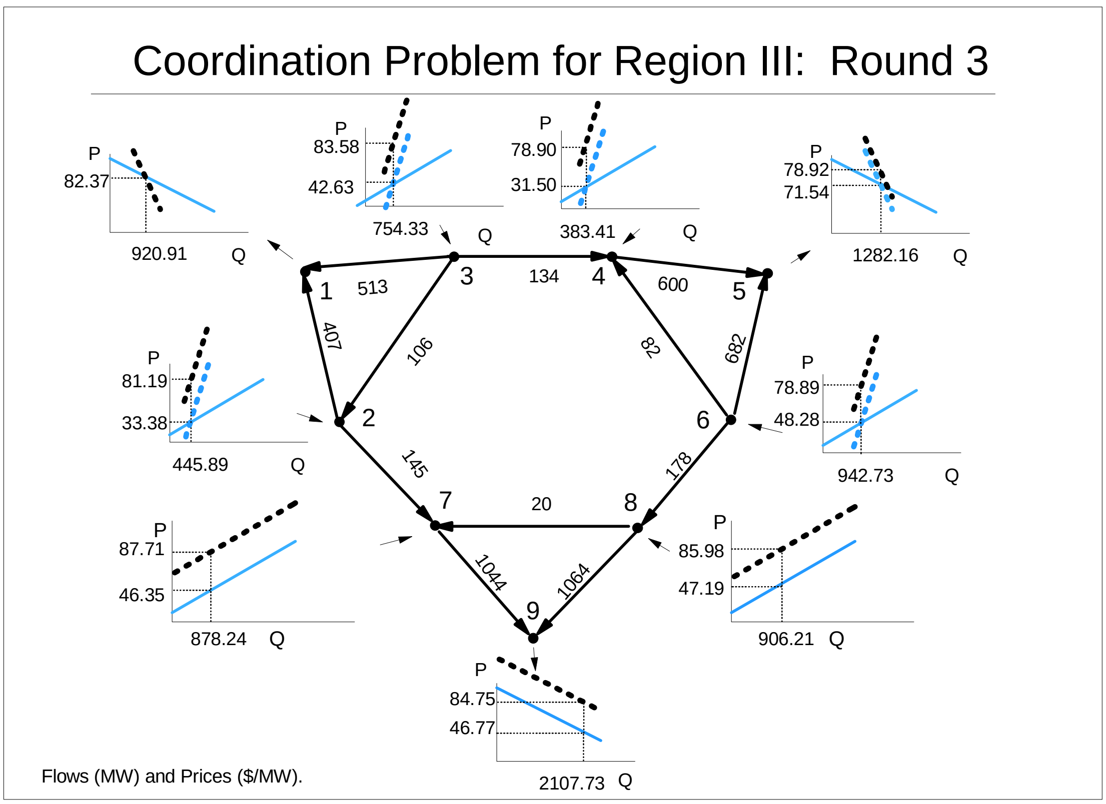
Again, the adjustments are most significant within region II, where the changes would have the most impact on the local constraints. However, the system-wide effects are important, generally calling for increases in load outside the region to help balance the limits on the line connecting location 4 and 5 inside region II.

The new solution also produces congestion costs for this now binding constraint in region II. These combine with the estimates in region I to produce a new set of total congestion costs for constraints in regions I and II, as in:

**Congestion Cost Estimates after Round 2**

| Bus | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| From I | 0 | 49.22 | 39.54 | 41.78 | 42.52 | 43.26 | 46.98 | 45.50 | 46.24 |
| From II | 0 | -1.41 | 1.41 | 5.62 | -35.14 | -12.65 | -5.62 | -8.43 | -7.03 |
| From III | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | | | | | | | | | |
| Outside I | 0 | -1.41 | 1.41 | 5.62 | -35.14 | -12.65 | -5.62 | -8.43 | -7.03 |
| Outside II | 0 | 49.22 | 39.54 | 41.78 | 42.52 | 43.26 | 46.98 | 45.50 | 46.24 |
| Outside III | 0 | 47.81 | 40.95 | 47.40 | 7.38 | 30.61 | 41.36 | 37.06 | 39.21 |

Apparently increasing supply at location 5 decreases congestion costs in region II but not as much as it increases relative congestion costs in region I. In the event, the congestion cost to be used by the system operator in region III is the aggregate change in external congestion costs for changing net load, or $7.38 per MW at location 5.




The results from round 2 now create a new coordination problem from the perspective of region III. This is the start of round 3, where the system operator in region III sees adjustment bids and congestion costs external to its region, and the original bids provided from the market participants within its region. This creates the corresponding coordination problem as shown in the accompanying figure.

As can be seen in the figure, the adjusted solution after round 2 is still not feasible in region III. Using the new estimates of net load in the combined schedules, adjustment bids relative to the new prices, and the new estimates of external congestion costs, the system operator in region III acts in turn in round 3 to redispatch the system to alleviate constraints within region 3, resulting in a revised set of adjustments and congestion costs. The resulting set of adjustments from round 3 would be:

**Round 3**

| **Location** | **Tentative Schedule** | **Adjustment** | **New Schedule** |
|---|---|---|---|
| 1 p | 82.37 | -0.08 | 82.30 |
| q | 920.91 | 2.55 | 923.47 |
| 2 p | 33.38 | 0.24 | 33.62 |
| q | -445.89 | -7.95 | -453.84 |
| 3 p | 42.63 | -0.39 | 42.24 |
| q | -754.33 | 13.08 | -741.26 |
| 4 p | 31.50 | -0.07 | 31.43 |
| q | -383.41 | 2.47 | -380.94 |
| 5 p | 71.54 | -0.22 | 71.32 |
| q | 1282.16 | 7.23 | 1289.39 |
| 6 p | 48.28 | -0.35 | 47.93 |
| q | -942.73 | 11.61 | -931.12 |
| 7 p | 46.35 | -2.32 | 44.03 |
| q | -878.24 | 77.32 | -800.91 |
| 8 p | 47.19 | -10.27 | 36.91 |
| q | -906.21 | 342.48 | -563.73 |
| 9 p | 46.77 | 13.46 | 60.23 |
| q | 2107.73 | -448.78 | 1658.95 |

The adjustments are still large within the region, and smaller outside. However, the same pattern is apparent. The external redispatch outside region III is significant and plays an important role in the overall solution.

The completion of one round for each region does not quite finish the process. Each region was making certain assumptions about the other regions, implicit in the estimate of congestion costs. However, the changes in schedules reverberate through all the regions, and the solution is not yet consistent across the regions. In other words, the adjustments in regions II and III have partly undone the adjustments in region I, again violating the transmission constraints in region I.

We return to region I, therefore, with a revised set of schedules of net loads and prices. In addition, region I now has the advantage of an estimate of the combined congestion costs from the constraints in regions II and III, as in:

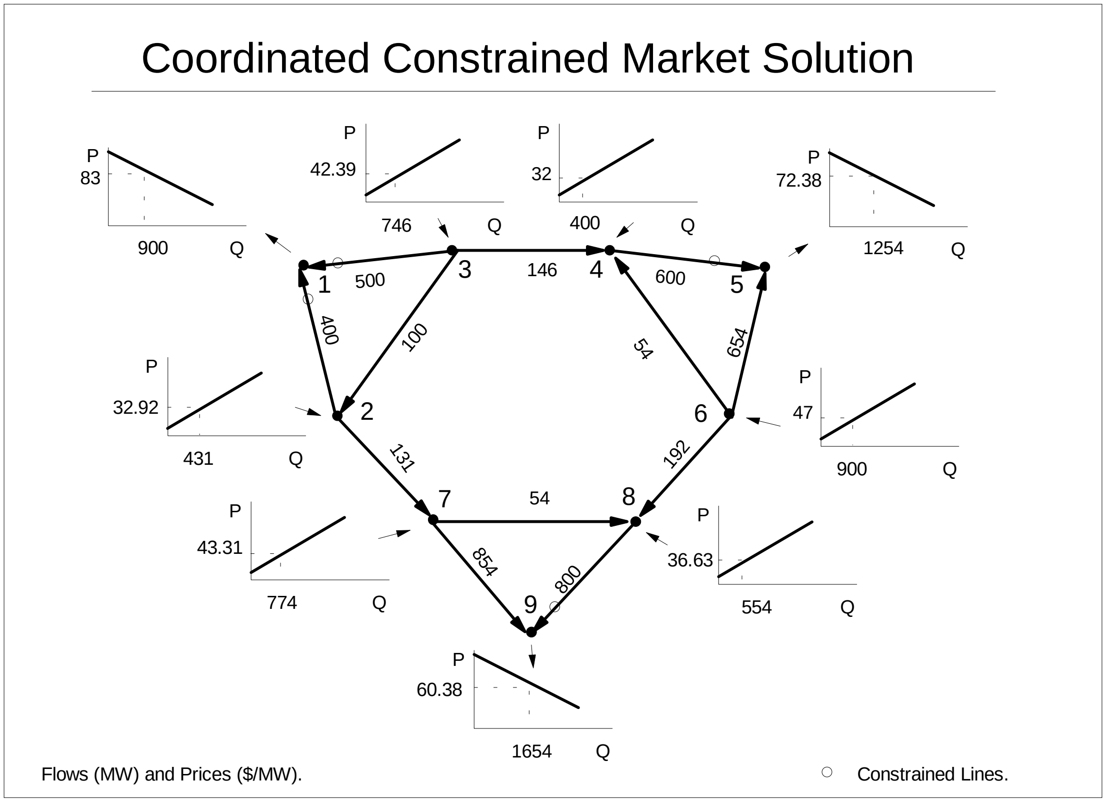
**Congestion Cost Estimates after Round 3**

| **Bus** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** |
|---|---|---|---|---|---|---|---|---|---|
| From I | 0 | 49.22 | 39.54 | 41.78 | 42.52 | 43.26 | 46.98 | 45.50 | 46.24 |
| From II | 0 | -1.41 | 1.41 | 5.62 | -35.14 | -12.65 | -5.62 | -8.43 | -7.03 |
| From III | 0 | -0.88 | 0.88 | 3.51 | 4.39 | 5.27 | -3.51 | 7.90 | -17.56 |
| | | | | | | | | | |
| Outside I | 0 | -2.28 | 2.28 | 9.14 | -30.75 | -7.38 | -9.14 | -0.53 | -24.59 |
| Outside II | 0 | 48.34 | 40.42 | 45.29 | 46.91 | 48.53 | 43.47 | 53.40 | 28.68 |
| Outside III | 0 | 47.81 | 40.95 | 47.40 | 7.38 | 30.61 | 41.36 | 37.06 | 39.21 |

Now region I sees the congestion costs outside I not as zero but as the corresponding values in the table obtained from the other regions. Using this information, the system operator in region I reformulates the system wide redispatch problem with its own constraints and the adjustment bids plus congestion costs from the other regions. It produces a new set of relative adjustments, and hands off responsibility to region II. And so on.

In principle, the process continues until there are no further efficient schedule adjustments needed and the schedules are fully consistent with all the regional constraints.

For this example, the process converged to the constrained solution as shown in the accompanying figure. This is the same solution obtained for the constrained network problem formulated as a single economic dispatch. It is a system-wide market equilibrium, and a region-by-region market equilibrium with coordinated congestion relief at the current prices.




The constrained locational prices range from a low of $32 per MW at bus 4 to a high of $83 at bus 1. There are four binding transmission constraints. In region I the lines between buses 1 and 2, and between buses 1 and 3, are at their limits. In region II, the line between buses 4 and 5 is constrained. In region III, the line between buses 8 and 9 is constrained.

These regional constraints give rise to the final coordinated estimates of congestion costs as shown in the accompanying table. Consider the "Outside I" row from the table, with the congestion cost at location 2 as $2.37 per MW. If the system operator from region I applies this congestion cost to its own redispatch calculation, and uses the corresponding estimate of the congestion costs at other locations, along with the final quantity schedules, it will conclude that it needs no further schedule adjustments. Furthermore, the system operator in region I would produce the congestion cost estimates for its constraints as shown in the row "From I."

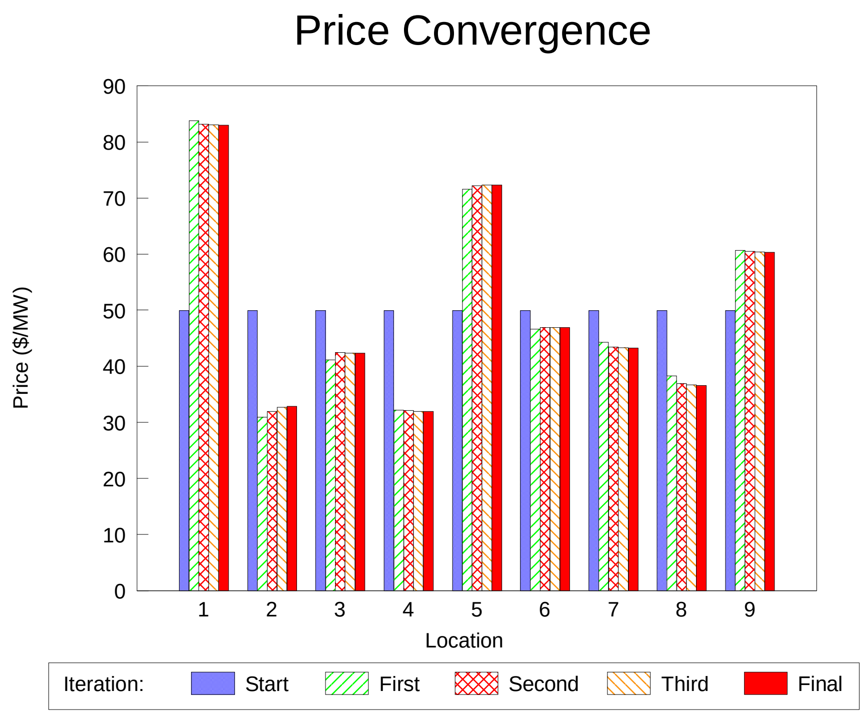
**Coordinated Congestion Cost Estimates**

| Bus | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| From I | 0 | 52.45 | 38.25 | 41.52 | 42.62 | 43.71 | 49.17 | 46.98 | 48.08 |
| From II | 0 | -1.46 | 1.46 | 5.85 | -36.54 | -13.15 | -5.85 | -8.77 | -7.31 |
| From III | 0 | -0.91 | 0.91 | 3.63 | 4.54 | 5.45 | -3.63 | 8.17 | -18.15 |
| | | | | | | | | | |
| Outside I | 0 | -2.37 | 2.37 | 9.48 | -32.00 | -7.71 | -9.48 | -0.60 | -25.46 |
| Outside II | 0 | 51.54 | 39.15 | 45.15 | 47.15 | 49.15 | 45.54 | 55.15 | 29.92 |
| Outside III | 0 | 50.98 | 39.71 | 47.37 | 6.08 | 30.55 | 43.32 | 38.22 | 40.77 |

This internal consistency would apply simultaneously to all the regions at this coordinated solution for congestion relief. Furthermore, since losses are not considered in the simplified example, the grand total of the congestion cost estimates across all three regions is just the locational price difference relative to the reference bus at location 1. For instance, the grand total for location 2 is $50.88 (52.45-1.46-0.91), which is precisely the difference in the prices at locations 1 and 2, or $50.88 = $83-$32.92.

Of course, all this detail need be seen only by the system operators in the process of working among themselves to arrive at a coordinated congestion relief solution. The situation would be much simpler from the perspective of the market participants who would see only the resulting dispatch and locational prices based on their schedules and market bids.

Convergence to the constrained solution for this problem is reasonably quick. Starting with the unconstrained answer, the following figure shows the resulting locational prices after each full iteration through all three regions. The prices adjusted almost to the constrained solution in the first full iteration through all the regions, and were essentially final after three iterations.




Price coordination iteration for nine bus example.

The convergence speed was essentially the same for an alternative test starting in region III and visiting the regional calculations in reverse order. One interesting feature of this alternative sequence was that the line between buses 5 and 6 was temporarily constrained during the first full iteration, but not afterwards.

The example problem network is looped and the regions are reasonably coupled.[7] In effect, power moving from one bus to another affects the flow on every line in the system.

It is not known if this same early convergence would extend to a realistic problem or full AC implementation. The issue of convergence and related experience is taken up further in the appendix. Obviously, the larger the regions and the weaker the coupling, the better should be the early convergence. As for the AC model, reactive power is by its nature most affected by local variables so the favorable convergence properties should be preserved. The related work of Kim and Baldick,[8] and similar applications on other economic equilibrium models, suggests that good early convergence may be a reasonable conjecture.

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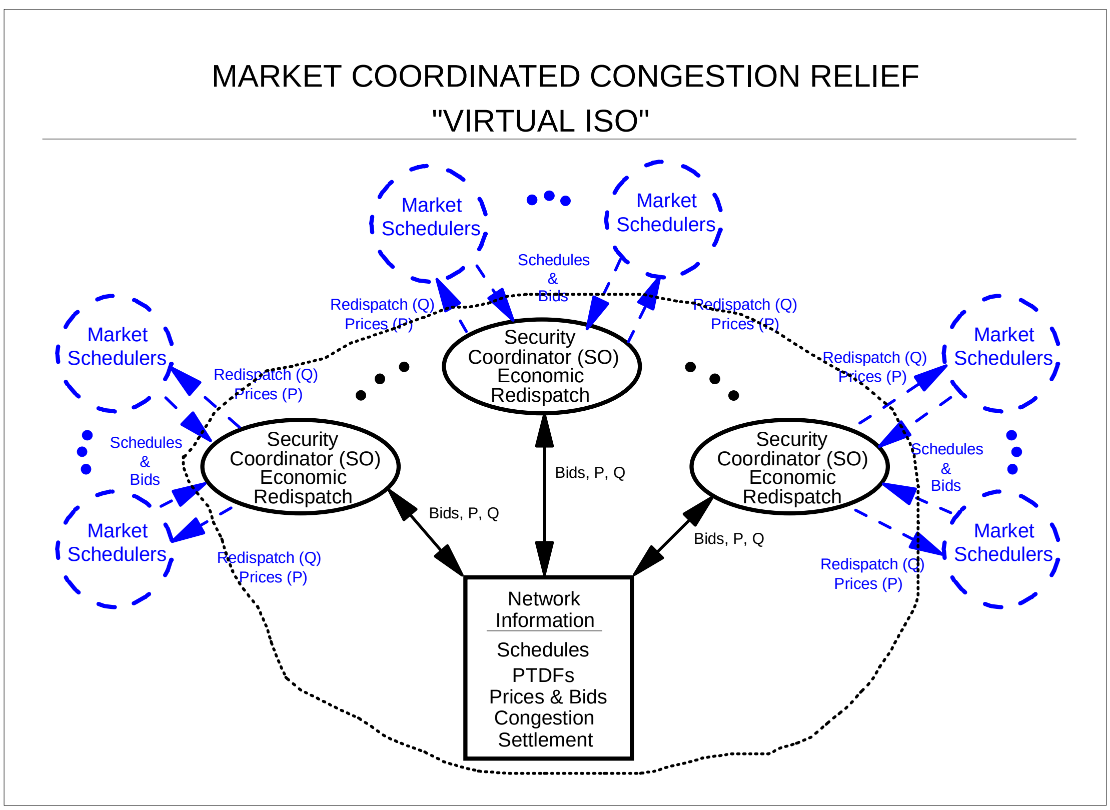
[7] The distribution factors relative to bus 1 are shown in the appendix.

[8] Balho H. Kim and Ross Baldick, "Coarse-Grained Distributed Optimal Power Flow," IEEE Transactions on Power Systems, Vol. 12, No. 2, May 1997, p. 937.

## INFORMATION EXCHANGE AND PAYMENTS

Coordination of multiple regions as envisioned here anticipates each region following certain rules and exchanging information. The coordination process does not quite require a coordinator. If each region follows the rules, the information need only be published on a bulletin board. However, reliance on market forces and coordination on prices requires that the prices provide meaningful incentives, so there would be payments made at the equilibrium prices.

The market rules within a region are not the focus here. For simplicity, assume that each market follows the same set of rules across the region. Hence, market participants have a deadline for submitting bilateral schedules and spot market bids. If the initial schedules are feasible, nothing more is required for congestion relief. Otherwise, the system operator uses this scheduling and bid information to find a bid-based, security-constrained, economic dispatch as illustrated above and described in more detail in the appendix. This requires both analysis of the dispatch within the region, and iterative exchange of information with the other regional system operators. The final equilibrium and associated prices are used in the settlement system. In effect, coordination among the system operators or regional security coordinators produces a "virtual system operator" for the entire system, as illustrated in the figure.



The basic regional model is familiar as an economic dispatch formulation in terms of the net loads. The process starts with an initial set of schedules ($y$), with adjustment bids $A(x)$, and with estimates of congestion prices ($\omega$) available for the entire grid. Given this information, each regional system operator solves its version of an economic dispatch and produces new estimates for its adjustment bids $A_j(x_j)$, for its congestion prices for the grid ($\omega_j$) and for its schedule adjustments for the grid ($x^j$). These in turn produce new estimates for the aggregate schedules, locational prices and locational congestion estimates. The process would continue further interaction among the system operators until an acceptable solution is obtained.

Compared to the initial TLR procedures adopted by NERC, the new information is predominantly in the form of bids and prices. The TLR transaction reporting system already requires explicit reporting of inter-regional transactions, and implicit estimation of the balance of the intra-regional schedules, in order to calculate the impacts on transmission constraints. The prices would be a new reporting requirement, but would seem to be essential in some form to implement a market-oriented coordination system. The system would not require one system operator for the entire grid. Rather, each regional system operator could start with the information obtained from the others to produce its needed update on the price effects induced by the constraints the particular system operator is monitoring. Given the prices, each system operator needs to keep track only of its own constraints.

The payments could take many forms. A natural organization would be to have users at each location treated as though they interacted with their local system operator at their respective locational prices. Those buying and selling in the spot market would use the equilibrium prices ($p$). Those scheduling bilateral transactions would pay the difference in the locational prices at the source and destination.[9] Some convention would apply as to which system operator would collect the payment for bilateral schedules that begin in one region and end in another. For instance, we could have the rule that the payment is always to the system operator at the destination. As usual, the system could operate in a hub and spoke framework, decomposing transactions between locations as being to and from the hub.

The settlement system in aggregate would have the usual property that net payments by loads would be at least as large as the payments to generators. In other words, we always have $py \geq 0$. In the case of active transmission constraints, the net payments would be positive, or $py > 0$. The difference would reflect the aggregate value of the transmission congestion.

This happy result that avoids revenue deficits in the aggregate would not be guaranteed for each region. Region by region, we could not determine the sign of $p_i y_i$, only that the sum across all regions would be non-negative. For instance, if one region had all the generation and another had all the load, there could not be individual payments

[9] An early implementation of this idea appears in the PJM proposal to allow transactions from outside the region to avoid TLR curtailments from PJM by paying congestion cost at the difference in locational prices. PJM Energy Committee Minutes of September 9, 1998, pp. D-9-7&8.

balance in each region. Hence, there would have to be a settlements system for the network as a whole. In other words, there would be payments to and from the various regions. The distribution of the surplus could be handled in various ways, such as in the creation of transmission congestion contracts as discussed in the appendix. [10]

## IMPLEMENTATION ISSUES

The conceptual framework for market coordination of congestion relief provides a guide to the development of the information and institutional arrangements. The framework allows for aggregation of regions and provides a path toward coordinated congestion relief for a market-based TLR arrangement providing coordinated congestion relief with "fewer people and more knowledge."[11] Essentially, congestion relief coordination would require information very much like that in the early NERC systems for TLR. The principal addition would be the adjustment bids and congestion cost estimates to be provided by the regional system operators or security coordinators.

Further consideration of the basic approach for coordination among regions would need to address a number of implementation issues and questions. Here we outline a few of the matters for future investigation.

### Drawing the Boundaries

Selection of the regions covered by the system operators will be governed by many factors, covering everything from the historical starting point to regional politics. However, to the extent the simplicity of coordination and efficiency of the market matter, the analysis here suggests an approach for further research in providing guidance for defining the boundaries of regional aggregations.

In the appendix, the formulation of the regional problem and the discussion of convergence results are both motivated by the analogy to and experience with iterative solution of "dominant diagonal" systems. The regional system operator gives explicit attention to the constraints within its region, but relies on more limited information in prices to capture the effect of constraints in other regions. If the constraints elsewhere have a small effect on the region's own prices, compared to the effect of the constraints within the region, then a simplified representation of the external effects should lead to a good solution within the region. The internalization of the most important constraints, therefore, creates the dominant diagonal condition that is so important in rapid convergence. By contrast, if the regional system operator did not have explicit knowledge of the effects of very important and sensitive constraints, it would seem that it

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[10] Scott M. Harvey, William W. Hogan, and Susan L. Pope, "Transmission Capacity Reservations and Transmission Congestion Contracts," Center for Business and Government, Harvard University, June 6, 1996, (Revised March 8, 1997).

[11] Paul McCoy suggested this summary statement of a goal for the TLR process.

would be more difficult to converge to a solution through iteration that relied only on the prices to capture the congestion effects.

The need to internalize the effects on constraints and the other requirements for coordinating ancillary services dictate that the individual regions should be at least large enough. The definition of large enough would depend upon the nature of the grid and the market. It is clear that an individual location would not be large enough, which is why a fully decentralized bilateral market is not feasible. There must be coordinators in the form of system operators. It is an open question as to whether existing control areas would be large enough, but not all control areas are the same and it is argued that there are too many control areas, suggesting aggregation would be preferred. The degree of aggregation would depend on many factors that go beyond the problems of coordination, and might be different in the various stages of development of the market. But the required regional aggregation for an efficient market system may not and need not extend to the entire grid and a grand coordinator.

The conjecture, therefore, is that a regional aggregation should be better when the interconnections are weaker in a particular sense. Not weaker in the sense that the connecting lines have only limited capacity, but weaker in the sense that the looped impacts across the boundaries are reduced relative to the looped impacts within the region. In the limit, obviously the best form of interconnection would be radial, where there would be no looped effects and no distant impacts on constraints and prices. The precise definition of weak loops is not clear, even in the simple example, but the goal is to have relatively little impact on the distant prices once a reasonable estimate of the prices is available.

**Regional Coordination Approximations**

Implementation of the protocol for coordinated congestion relief would confront a number of modeling approximations and transition issues. For example, as discussed at further length in the appendix, the approach requires each regional system operator to have available a description of the full network. The illustrative example assumed that every system operator uses the same description of the network. Each regional system operator need monitor only its own constraints, but it does need to determine the flows in the network.

This is no different than today, with each system operator maintaining an "equivalenced" description of the full network. The detail in these models is greatest within the region, and the detail of the flows outside the region is reduced through equivalent aggregation. This induces an approximation error in the estimate of the flows. However, it would not severely affect the coordination protocol. Just as today, the important feature of each regional model is that the equivalenced network provide an acceptable approximation of the impact of external loads on internal constraints. Errors in the determination of the flows on the external lines are important only to the extent that they affect the estimates of power flows on internal lines. This, of course, has always been true for monitoring system operations, and the approximations required for normal operations carry over directly for the process of coordinated congestion relief. The coordination problem does not introduce any new demands on the modeling of the network.

A related issue would arise in case some of the regions were not cooperating in the coordination protocol. At a minimum during a period of transition, it should be assumed that some of the regions are participating in coordinated congestion management, and others are still following non-market, administrative curtailments. Notwithstanding the fact that it would be better to take advantage of the market, the coordination protocol should not depend on full cooperation before anything could be done. Again there are well known methods for dealing with external constraints that must be addressed even when the local system operator is not following the market based protocol. In effect, therefore, some regions could follow a NERC administrative TLR process and other regions could participate in coordinated congestion relief. The appendix outlines the details. The basic conclusion suggests that it would be possible to begin a coordination process that involved a small number of regions, working to the advantage of the market participants within those regions, and gradually expanding to include new regions in the coordination process. In principle, the expanded coordination could extend to the entire interconnected grid without requiring consolidation of the many details currently under the responsibility of individual control regions.

### Gaming and Honest Revelation

The coordination framework outlined above assumes that it is possible to find trial solutions and get meaningful estimates of the required schedule adjustments. In effect, the method assumes that it is possible to get the system operators and, by implication, the market participants to give honest answers for use in the iteration process. We are assuming that the regional system operators do not game the process by providing misleading estimates or strategic changes in the proposed schedules.

For the case of the regional system operators that follow the model of bid-based, security-constrained, economic dispatch, this would be a reasonable assumption. We would, of course, still have whatever limitations there are in strategic bids of the market participants. But this would be another matter. Once the bids were provided, the system operators would be making the decisions needed to resolve the regional coordination and congestion relief issues based on the bids provided in advance by the market participants. The system operators would be charged to seek an efficient solution as far as the coordination process goes, just as they are charged to seek an efficient dispatch. The market participants would not be asked to respond to the interim prices during the coordination iteration, so they would not be in a position to game the coordination process through strategic revelation of information.

This resolution of gaming problems would not be available for other market models that would give the market participants an opportunity to respond to the interim coordination price and congestion information. In effect, we would be involved in an iterative auction where the participants would not necessarily face any consequences for strategic schedules later withdrawn. Hence, for market models without regional dispatch by a system operator, there would have to be some rules designed to make interim bids meaningful and to move the market efficiently towards a solution. This could be a challenge. The complex network interactions make simple activity rules and bidding constraints inappropriate for the electric network coordination problem. For example, requiring monotonic bids from the market participants, a common feature of such rules in other settings, would not be consistent with the coordination process here. As we can see even from the simple example, the prices do not move monotonically towards the solution, and the network interactions can have substantial indirect impacts on the efficient prices.

In the coordination process described here, therefore, the complications of the iterative process are encapsulated in the coordination among regional system operators, which is essentially a solution procedure and not an iterative market. It remains to be seen if any method could be devised for regional coordination within the framework here that allowed for consideration of customer preferences through anything other than the standard model of the regional system operator running a bid-based economic dispatch.

## Long-Term Transmission Rights

Long-term transmission rights could be defined in the usual way as financial rights to the difference in location prices or congestion costs.[12] For simplicity assume that we focus on congestion costs in a DC-load model formulation without losses. Then all the differences in locational prices would be attributable to congestion costs. Transmission congestion contracts (also known as Fixed Transmission Rights, or Financial Congestion Rights, or Financial Transmission Rights) would be available to collect the difference between the congestion costs at two locations.

In one interpretation, the transmission congestion contract could be allocated to market participants in a combined process that involved all the system operators. In this case there would be nothing new or special about the problem. In order to guarantee revenue adequacy, i.e., that the actual collection of congestion payments would be sufficient to fund the payments under the transmission congestion contracts, the test of simultaneous feasibility would apply. But now the simultaneous feasibility would include all the regional constraints. An auction of a simultaneously feasible allocation of transmission congestion contracts could be arranged through a slightly modified application of essentially the same coordination mechanism to regional transmission congestion auctions. This would exploit the fact that transmission congestion contract auctions have the same basic form as an economic dispatch problem. Collections and payments for the transmission congestion contracts would be made through an auction settlement system.

An alternative approach that is more in keeping with a "separate but coordinated" spirit would have each regional system operator allocate transmission congestion

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[12] Scott M. Harvey, William W. Hogan, and Susan L. Pope, “Transmission Capacity Reservations and Transmission Congestion Contracts,” Center for Business and Government, Harvard University, June 6, 1996, (Revised March 8, 1997).

contracts that would be feasible in its region without consideration of any external constraints. In this case, there would be no need to check for simultaneous feasibility across all regions. The rights would be defined between any two points in the grid. There would be no necessity for the separate right configurations to be consistent. In exchange, however, the payments under the transmission congestion contracts would be limited to the congestion costs arising from that same region's internal constraints alone.

As discussed in the appendix, this separate allocation of transmission congestion contracts would still be revenue adequate. However, if the rights were not consistent across regions, then some of the market participants would remain exposed to the congestion costs arising from the regions where they had not obtained an equivalent transmission congestion contract. In other words, if there are three regions and three system operators, for these decentralized rights to provide complete protection between any two points, it would be necessary for the market participant to get three transmission congestion contracts, one from each system operator. Of course, if everyone preferred such complete protection, then the transmission congestion contracts would necessarily be consistent across regions and we would default to the coordinated allocation as above.

In concept, it would be possible to arrange both types of transmission congestion contracts, or any combination of the basic building blocks. The only important issue for ensuring the revenue adequacy of the rights would be to maintain internal consistency with the principle that the rights should be simultaneously feasible for all the constraints to which they apply.

**Computational Requirements**

Part of the motivation for coordination approaches is that the methods would get close to a solution relatively quickly. Most real system operator processes will require some human intervention. The real security-constrained economic dispatch problem is too complicated to be fully automated. And without virtually complete automation, it would be of little use to have a coordination method that depended on hundreds or thousands of iterations to get reasonable answers.

As discussed further in Cadwalader et al.,[13] from the perspective of convergence analysis, we want the adjustment method to behave like a contraction mapping. In other words, at each step we want substantial movement towards the solution. The details of the problem here are complicated, and the network interactions challenge our ability to be sure that the methods will converge at all, much less rapidly. However, our intuition is that the system should behave much like the analogous iteration methods on diagonal dominant systems of equations. If each region’s principal impact is on its own prices and constraints, being not as highly coupled with the prices and constraints in other regions, we would expect the most important changes in the critical price variables to occur as a

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[13] Michael Cadwalader, Scott Harvey, William Hogan, and Susan Pope, "Market Coordination of Transmission Loading Relief Across Multiple Regions," Center for Business and Government, Harvard University, December 1, 1998.

result of the local regional solution. If the price variables for other regions don't change as much, then there should be rapid convergence.

An important source of support for this conjecture is found in the related work of Kim and Baldick.[14] They developed an innovative representation of the network and applied decomposition methods. The details are important, but at a certain level of abstraction the basic iterative method has much in common with the proposal outlined above. Kim and Baldick implemented their approach and reported exactly this type of favorable convergence experience for their large test networks with specialized software for parallel computation. Their focus was on the implications for distributed computing, but their results have implications for market coordination. In their work, "the most significant feature … is that the solution converges within 3 or 4 iterations." Whether or not this happy result would extend to larger, more realistic problems is an open question, but their experience gives reason for optimism.[15]

The coordination method proposed in the present paper was designed to work with the existing method of characterizing the network. Hence, it would be amenable to computational testing on a larger scale with existing software. For instance, consider a large scale model like GE MAPS, that has an extensive description of the network, regional dispatch, and contingency constraints. It would be conceptually straightforward to simulate the coordination protocol across multiple regions with the full description of the grid. Suppose, for sake of discussion, we considered three regions. Then in effect we would need three versions of MAPS, one to represent the problem as seen from each region. There would have to be some development of a data transfer protocol, to simulate the exchange of information among system operators. This would require an investment, but is not a major undertaking and users of MAPS are familiar with the software requirements.

Given access to MAPS and creation of the data transfer software, the exercise of this large model to simulate coordinated congestion relief should not be more difficult than simulating several market solutions with MAPS. Not cheap, but cheap compared to the alternative.

Assuming the basic method proves workable and robust within the framework of a modeling environment where everything is under control, it would be reasonable to test the idea using market simulators. Various system operators have been developing and using simulators to test market designs and institutions. The extension to using these simulators for coordinated congestion relief would be relatively modest. Data on market participant bids and schedules could be taken directly from existing simulations or even

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[14] Balho H. Kim and Ross Baldick, "Coarse-Grained Distributed Optimal Power Flow," IEEE Transactions on Power Systems, Vol. 12, No. 2, May 1997, p. 937.

[15] The method of Kim and Baldick has been tested on a 2587 line representation of ERCOT by Baldick et al., apparently without contingency constraints. There is no report of the number of iterations for this problem. However, they do describe substantial computational efficiencies measured in elapsed time which should imply rapid convergence in terms of the number of iterations: Ross Baldick, Balho H. Kim, Craig Chase and Yufeng Luo, "A Fast Distributed Implementation of Optimal Power Flow," (To appear in IEEE Transactions on Power Systems).

live market information. Since the coordination process is by design limited to communication among the system operators, they could use the data and the simulators to test the coordination process. This would not involve anything like the complexity of building the simulators from the ground up to deal with the entire market. The only new feature required would be the reporting mechanism that provided the data transfer for prices, net loads and adjustment bids. The necessary settlement system could come later, and need not be part of the simulation of coordinated congestion relief.

## CONCLUSION

Coordination of congestion relief across a very large grid may not require a grand coordinator, as long as each individual region with its own system operator is large enough to internalize the primary effects of its own transmission constraints. The coordinated congestion relief protocol outlined here provides a method with rules for exchange of information among system operators in an interconnected grid. If the regional system operators follow the rules, an aggregate solution would yield an equilibrium market-based result for coordinated congestion relief across the entire grid. The method requires nothing revolutionary. Each system operator would apply conventional methods for redispatch using congestion cost and redispatch information from the other regions. Any two regional system operators could begin the process, later adding other regions to expand the scope of coordinated congestion relief. The theory, simple examples, and experience from other applications, all suggest that the coordination method should be quite effective. A larger scale test would be useful to consider the application for the real network.

## APPENDIX

Here we outline the underlying details of the congestion relief model and the coordination protocol.

## MARKET MODEL WITH FULL COORDINATION

With the goal of describing coordination among system operators, it is convenient to begin with a market equilibrium that is equivalent to an economic dispatch formulation with full optimization across the entire system, but which explicitly recognizes the existence of multiple regions. For this purpose, we define a model of the power system and a bid-based, security-constrained, economic dispatch.

Let:

| | |
|---|---|
| $y_i$ | the vector of net loads at the buses in region i, equal to demand minus generation at each bus, for regions i = 1, 2, …, n, |
| $B_i(y_i)$ | the bid-based net benefit function for net loads in region i, |
| $L(y_1,\ldots,y_n)$ | the system constraint on net loads at all buses to ensure balance with losses and generation, |
| $K_i\,(y_1,\ldots,y_n)$ | the vector of constraints in the transmission grid in region i. |

The net loads could be interpreted as for both real and reactive power in a full AC formulation of the optimal power flow or economic dispatch problem. However, nothing would be lost from the interpretation below if we think of the model in terms of real power only.

The regional net-benefit function $B_i$ represents the benefits of load minus the costs of generation at each bus aggregated for the region. We can think of this as constructed in the usual way from the upward sloping supply bids and downward sloping demand bids of the market participants at each location.[16] Bilateral transactions would be included in the usual way as fixed schedules with or without increment and decrement bids that would be part of the benefit function. The rest of the discussion does not require any further explicit consideration of bilateral schedules, other than to recall that that the gross total payments under the system will depend in part on the volumes that flow through the spot market. The net payments for losses and congestion rents would not be affected by the inclusion of bilateral transactions.

[16] Hence the net-benefit function is concave and separable across regions. For simplicity, we assume that the function is differentiable, but this could be relaxed to include step functions without affecting the discussion conclusions here.

The load balance constraints in $L$ include the system wide requirement to balance loads, generation and losses. In the AC model there would be two elements in the vector, for real and reactive power. In a real-power only approximation, this would be the single load balancing constraint.

The constraints $K_i$ include all the possible limitations on the flow of power in the grid, including thermal, voltage, stability, or any other limits.[17] The constraints are represented here as a function of the net loads at each bus. The formulation treats other variables, such as voltage magnitudes and angles, as intermediate values that are implicit in the problem but suppressed in the explicit model formulation. The transmission constraints include all limits that would arise in the event of a set of monitored contingencies. The number of constraints included in $K_i$ could be quite large, but as is usual in these matters, the ultimate focus will be on the binding constraints in the ultimate dispatch.

This approach to modeling the constraints is referred to as a full grid model in that the explicit variables are only the net loads.[18] Hence, the model of constraints must have a characterization of the full grid, and it is assumed possible to determine the impact on any constraint from net load at any bus. Details in the form of the model may differ among regions. For instance, the choice of a reference bus for the network equations is arbitrary, as long as it is consistent within a region. Likewise, we assume that the regional system operator knows about the contingencies and corresponding constraints that apply for its region, but may have little or no information about the constraints in other regions. The purpose of the coordination process is to provide for an information exchange among the regions without requiring each region to solve the entire problem.

Under the usual assumptions, a market equilibrium gives rise to an economic dispatch. The basic formulation of the problem that we would like to solve can be summarized as the same as the bid-based, security constrained, economic dispatch over the entire grid. The representation is at a high level of abstraction to emphasize the important details of the coordination problem. However, the representation is consistent with standard economic dispatch procedures. For a given dispatch hour, we choose the net loads to maximize the total sum of the net benefits over the entire grid:

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[17] The constraints could be many and complex, driven by the effects of Kirchoff's laws on power flows. As usual, we assume that the resulting constraint functions are differentiable

[18] For a discussion of alternative formulations, see Michael Cadwalader, Scott Harvey, William Hogan, and Susan Pope, "Market Coordination of Transmission Loading Relief Across Multiple Regions," Center for Business and Government, Harvard University, December 1, 1998. In particular, for a related approach applies a different from of decomposition that is less like standard economic dispatch procedures, see Balho H. Kim and Ross Baldick, "Coarse-Grained Distributed Optimal Power Flow," IEEE Transactions on Power Systems, Vol. 12, No. 2, May 1997, pp. 932-939.

$$
\begin{aligned}
&\underset{y_1,\cdots,y_n}{Max} \quad \sum_{i=1}^{n} B_i(y_i) \\
&subject\ to \\
&L(y_1,\cdots,y_n)=0\ , \\
&K_i(y_1,\cdots,y_n)\le 0, \quad i=1,2,\cdots,n.
\end{aligned}
\tag{1}
$$

A solution to this optimization problem would give rise to constraint multipliers and a vector of locational market prices for each region that would satisfy the relation:[19]

$$
p_i=\nabla B_i=\theta\nabla L_i+\sum_{j=1}^{n}\lambda_j\nabla K_{ji}\ . \tag{2}
$$

Here the gradient $\nabla L_i$ is the marginal impact on generation and losses of an increase in loads at the buses in region i, and $\theta$ has an interpretation as the price of power at the reference bus selected for the load flow calculations. The matrix of gradients in $\nabla K_{ji}$ captures the impacts on the j$^{th}$ region's constraints from an increase in the loads of the buses in the i$^{th}$ region, and the variables $\lambda_j$ represent the constraint prices or marginal values of the transmission limit. Note that most constraints will not be binding. Hence, $\lambda_j$ will be zero for most constraints, excepting the binding constraints.[20]

In the terms of the Transmission Loading Relief (TLR) procedures, the gradients $\nabla K_{ji}$ are the distribution factors for the constraints assuming the change in net loads at the bus is balanced at the reference bus. In general, these distribution factors depend on the configuration of the net loads and the configuration of the network. In concept, the equivalent NERC Power Transfer Distribution Factors (PTDF) for any other transaction between locations would equal the difference in the corresponding elements of $\nabla K_{ji}$. Hence, these critical data can be available and have a familiar interpretation. In practice, the NERC procedures for TLR updating do not change the distribution factors for each dispatch, and to the extent that PTDF are changing a more dynamic system as envisioned here would be indicated.

## REGIONAL DECOMPOSITION

There is a close connection between the binding transmission constraints and the constraint prices. Anticipating the decomposition of the problem by regions, we

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[19] For the real power case with the usual DC-load approximation, see F. C. Schweppe, M. C. Caramanis, R. D. Tabors, and R.E. Bohn, Spot Pricing of Electricity, Kluwer Academic Publishers, Norwell, MA, 1988. M. C. Caramanis, R. E. Bohn, and F. C. Schweppe, "Optimal Spot Pricing: Practice and Theory," IEEE PAS, Volume PAS-101, No. 9, September 1982, develops optimal spot pricing for both real and reactive power.

[20] For a discussion of dispatch-based pricing, see William W. Hogan, E. Grant Read and Brendan J. Ring, "Using Mathematical Programming for Electricity Spot Pricing," Energy Models for Policy and Planning, International Transactions of Operational Research, Vol.3, No. 3/4, 1996.

recognize that for any region we could focus on the transmission constraints it monitors and "price out" the constraints in other regions. In terms of optimization theory, this is a selective dualization of the problem.[21] Hence, for region j it follows that if we know the constraint prices for the other regions (i≠j), a solution for the economic dispatch problem in (1) would also be a solution for the dualized problem:[22]

$$
\begin{aligned}
&\underset{y_1,\cdots,y_n}{Max} \quad B_j(y_j) + \sum_{i \neq j} B_i(y_i) - \sum_{i \neq j} \lambda_i K_i(y_1,\cdots,y_n) \\
&\textit{subject to} \\
&L(y_1,\cdots,y_n) = 0\,, \\
&K_j(y_1,\cdots,y_n) \leq 0\,.
\end{aligned}
\tag{3}
$$

The choice of region is arbitrary. Furthermore, in the context of regional coordination it is natural to think of the existing schedules as given and emphasize the changes in the schedules to achieve redispatch. Then we can restate (3) by viewing the net loads ($y$) as given and formulate the problem as the determination of the deviations ($x$) from the given schedules, as in:

$$
\begin{aligned}
&\underset{x_1,\cdots,x_n}{Max} \quad B_j(y_j + x_j) + \sum_{i \neq j} B_i(y_i + x_i) - \sum_{i \neq j} \lambda_i K_i(y_1 + x_1,\cdots,y_n + x_n) \\
&\textit{subject to} \\
&L(y_1 + x_1,\cdots,y_n + x_n) = 0\,, \\
&K_j(y_1 + x_1,\cdots,y_n + x_n) \leq 0\,.
\end{aligned}
\tag{4}
$$

If the given loads and constraint prices in (4) are an optimal solution to the economic dispatch problem in (1), then an optimal solution for the deviations would be zero.

Our focus on market solutions and prices motivates another reformulation through linearization of the problem. We apply this linearization in terms of the deviations ($x$). For instance, we have

$$
K_k(y_1 + x_1,\cdots,y_n + x_n) \approx K_k(y_1,\cdots,y_n) + \sum_i \nabla K_{ki} x_i\,. \tag{5}
$$

---

[21] Arthur M. Geoffrion, "Duality in Nonlinear Programming: A Simplified Applications-Oriented Devleopment," SIAM Review, Vol. 13, 1971, pp. 1-37.

[22] Shmuel Oren noted that the argument is motivated by the completely convex case, such as with the DC-load model for real power. In the AC-model, even without global convexity, the same argument applies to the solution of the first order Kuhn-Tucker necessary conditions of an optimum. The constrained AC problem could be well behaved in the sense that a solution for the first order conditions provides a solution for the market equilibrium problem. If not, the difficulties would extend beyond the mechanics of decomposition to call into question the existence of a competitive market equilibrium and might point to a greater role for more direct management of the grid and less reliance on markets.

With this linearization, we can restate the objective function in terms of the deviations in the constraints in other regions, as in:

$$\begin{aligned}
&\underset{x_1,\cdots,x_n}{Max} \quad B_j(y_j + x_j) + \sum_{i \neq j} B_i(y_i + x_i) - \sum_i \sum_{k \neq j} \lambda_k \nabla K_{ki} x_i \\
&\textit{subject to} \\
&L(y_1 + x_1, \cdots, y_n + x_n) = 0\,, \\
&K_j(y_1 + x_1, \cdots, y_n + x_n) \leq 0\,.
\end{aligned} \tag{6}$$

We have dropped the constant terms from the objective function. Here we assume that the system operators in each region are providing the bid information from their region for use in determining the adjustments to achieve a feasible solution. However, the information about constraints is contained only in the linearization of the corresponding term in the objective function. If the constraint prices and net loads *y* in (6) provide a competitive market equilibrium and, therefore, a solution to the economic dispatch problem, then again zero would be an optimal solution for the deviations in this linearized problem.

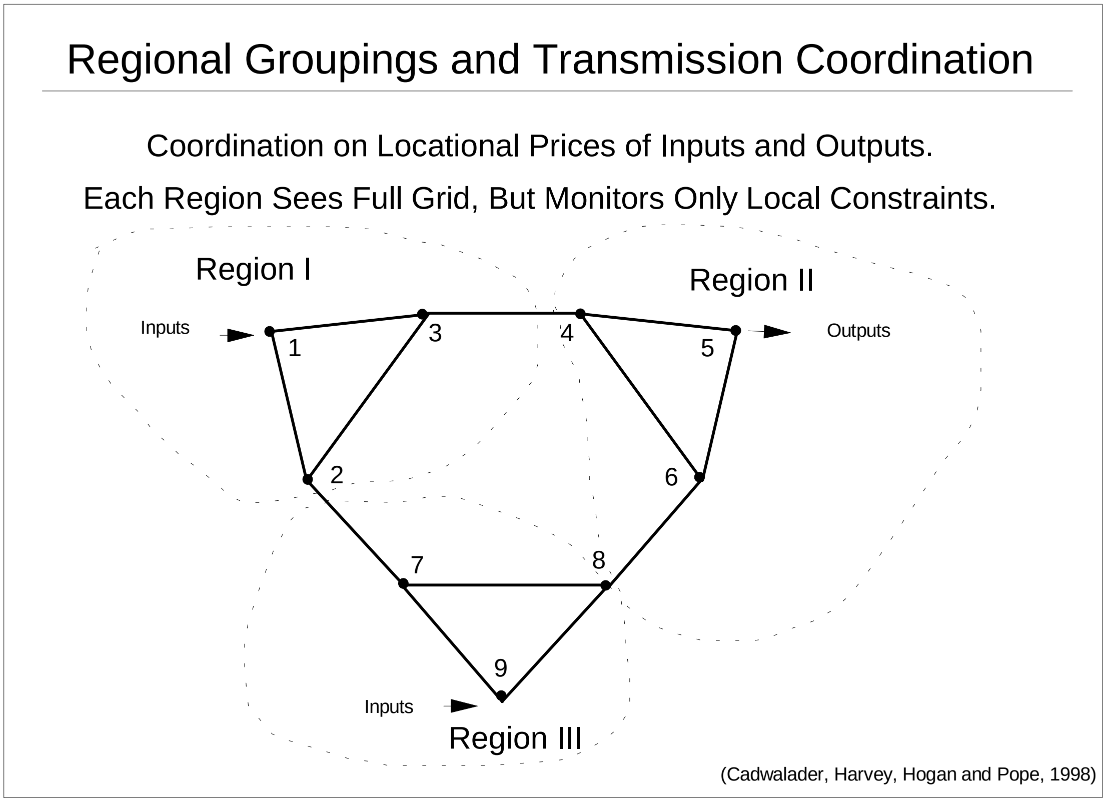
An alternative way to write the linearized problem would be in terms of the market locational congestion components of the prices, $\omega_{ki} = \lambda_k \nabla K_{ki}$, for an increment in the net load. In other words, $\omega_{ki}$ is the marginal congestion opportunity cost for loads in region i induced by the constraints in region k. Given these individual congestion cost estimates induced by the constraints in each region, we can aggregate to an estimate of the congestion cost for constraints outside region j as in:

$$\varpi_{ji} = \sum_{k \neq j} \omega_{ki}\,. \tag{7}$$

Then (6) becomes:

$$\begin{aligned}
&\underset{x_1,\cdots,x_n}{Max} \quad B_j(y_j + x_j) + \sum_{i \neq j} B_i(y_i + x_i) - \sum_i \varpi_{ji} x_i \\
&\textit{subject to} \\
&L(y_1 + x_1, \cdots, y_n + x_n) = 0\,, \\
&K_j(y_1 + x_1, \cdots, y_n + x_n) \leq 0\,.
\end{aligned} \tag{8}$$

The accompanying figure illustrates the regional decomposition.




This formulation of the problem lends itself to a natural interpretation.

The formulation in (8) takes the perspective of an arbitrarily selected region. The function $B_j$ defines the local net benefits based on the bids. The term $\varpi_{ji}x_i$ is the congestion payment to other regions for the change in net loads in region i. Hence, given the net load nominations made by all the other regions, and given the market clearing congestion prices ($\varpi$) elsewhere, the local problem for the regional system operator is to choose the redispatch deviations from the nominations across the grid, subject to load balancing and local constraints, so as to maximize the net benefits minus the cost of congestion. This still has the form of an economic dispatch problem. It is, for example, the type of problem solved by the PJM ISO, ignoring the congestion costs in the external regions. However, including these congestion costs would be easy as another form of bid function, if they were available, and would not require any fundamental reformulation of the problem.

We could formulate this problem for each region. Each region would give explicit attention to its local constraints, but not to the constraints of other external regions. The congestion prices would capture the effects of external constraints. If the external prices and nominations are at the market equilibrium, then each region would have zero deviation as an optimal solution for its version of (8).

Given arbitrary nominations $y$ and estimates of the market prices $\omega$, the corresponding statement of the adjustment problem could now distinguish the solution as seen from the perspective of each region. Hence, if we identify $x^j$ as the full system adjustment vector as seen from region j, $x^j = (x_1^j, \ldots , x_n^j)$, then we could restate (8) as in:

$$\begin{aligned}
&\underset{x_1^j,\cdots,x_1^j}{Max} \quad B_j(y_j + x_j^j) + \sum_{i \neq j} B_i(y_i + x_i^j) - \sum_i \varpi_{ji} x_i^j \\
&\textit{subject to} \\
&L(y_1 + x_1^j, \cdots, y_n + x_n^j) = 0\,, \\
&K_j(y_1 + x_1^j, \cdots, y_n + x_n^j) \le 0\,.
\end{aligned} \tag{9}$$

Heuristically, we could imagine a coordination process that used (9) to generate a sequence of adjustments in the system nominations. One approach would be to update each region in turn. Another sequence would be to allow all the regions to update simultaneously.[23] Hence, we would start with an estimate of $y$ and $\omega$. Each region would solve its version of (9) to generate its adjustment vector $x^j$. For the simultaneous update, the new estimates might be:

$$\begin{aligned}
&y^{new} = y^{previous} + \sum_{j=1}^{n} x^j / n\,, \\
&\omega^{new} = (\omega_1^{new}, \cdots, \omega_n^{new})\,,\ with \\
&\omega_j^{new} = (\omega_{j1}^{new}, \cdots, \omega_{jn}^{new}) = (\lambda_j^{new} \nabla K_{j1}(y_1^{previous} + x_1^j, \cdots, y_n^{previous} + x_n^j), \\
&\qquad\qquad \cdots, \lambda_j^{new} \nabla K_{jn}(y_1^{previous} + x_1^j, \cdots, y_n^{previous} + x_n^j))\,.
\end{aligned} \tag{10}$$

The motivation here is that a region's decisions should have the principal impact on its estimate of the implied cost of congestion for the constraints monitored by the region. The revised estimates of the local congestion prices and system schedules would provide more and better information for all the other regions.

If this process converges, then we would have a competitive market equilibrium. This approach is similar to the market process outlined by Schweppe et al., in which a single auctioneer would update prices and the market participants would respond to these prices.[24] The coordination process here is different than that proposed by Schweppe et al., however, in that there is not just a single auctioneer (i.e., not just one system operator) announcing prices. Furthermore, the responding regions internalize some of the constraints to produce new estimates of the congestion prices, not just new estimates of

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[23] This is analogous to an application of Gauss-Seidel or Jacobi iteration on a set of non-linear equations. See J. M. Ortega and W. C. Rheinboldt, Iterative Solution of Nonlinear Equations in Several Variables, Academic Press, 1970.

[24] F. C. Schweppe, M. C. Caramanis, R. D. Tabors, and R.E. Bohn, Spot Pricing of Electricity, Kluwer Academic Publishers, Norwell, MA, 1988.

the net loads. A slightly more subtle point is that iteration through the regions guarantees that an overall solution would satisfy not only the first-order necessary conditions for optimality, but also the complementarity requirement that constraint prices are zero for non-binding constraints and positive only when the constraints are binding.

A slightly modified version of this problem would call for the adjustment bid function information provided by the system operators to apply only to the deviations from the current schedules. Suppose that given the tentative schedules, $y$, we define $A_i(x_i) \approx B_i(y_i + x_i) - B_i(y_i)$, with $\nabla A_i = \nabla B_i = p_i$. Then a more suggestive notation for the problem would have each adjustment cycle based on the problem:

$$\begin{aligned}
&\underset{x_1^j,\cdots,x_1^j}{Max} \quad B_j(y_j + x_j^j) + \sum_{i \neq j} A_i(x_i^j) - \sum_i \varpi_{ji} x_i^j \\
&\textit{subject to} \\
&L(y_1 + x_1^j, \cdots, y_n + x_n^j) = 0\,, \\
&K_j(y_1 + x_1^j, \cdots, y_n + x_n^j) \leq 0\,.
\end{aligned} \tag{11}$$

Again, if the constraint prices and net loads $y$ in (11) provide a competitive market equilibrium and, therefore, a solution to the economic dispatch problem, then again zero would be an optimal solution for the deviations in this linearized problem for determining the coordination solution for adjustments across the grid.

We can think of (11) as defining the final version of the modified regional economic dispatch problem. The system operator knows its own aggregate bid function and the current schedules of net loads throughout the grid. In addition, it has estimates of the congestion cost prices for all the other regions, $\varpi$, and the adjustment bids at the locations identified by the other regions, $A(x)$. Both of these latter elements, the external congestion price estimates and the adjustment bids, are just special cases of elements of the objective function for a security-constrained, bid-based, economic dispatch.

The role of the external congestion cost estimates, $\varpi$, is to shift the apparent price in the supply or demand underlying the benefit functions or adjustment bids, as discussed below. In effect, therefore, (11) is a problem of exactly the same form as the familiar economic dispatch problem. Expanding the spot market protocol to solve this problem would require nothing more than including some adjusted bids in the model. And the adjustments are rather simple.

## DEFINING ADJUSTMENT BIDS

The regional system operators have flexibility in the choice of the form of the adjustment bids. The aggregate benefit function in terms of net loads is convenient for certain analytical purposes. However, this simplification hides important details that can be relevant in coordinating the market and in describing the process to the participants. Here the model is unpacked slightly to set the stage for discussion of alternative adjustment bid approaches within regions.

The aggregate benefit function derives from the underlying demand and supply bids in the usual way. In the most general case, we have demand and supply bids from the market participants, in the form of:

| | |
|---|---|
| *p(d)* | the vector inverse demand function, separable across buses; |
| *mc(g)* | the marginal costs for generation, also separable across locations; |

then we can define the benefit and cost functions as:

$$B^{*}(d)=\int_{0}^{d}p(x)dx\,,$$
$$C(g)=\int_{0}^{g}mc(x)dx\,. \tag{12}$$

Here it is convenient to assume that these benefit and cost functions are differentiable. In actual implementation the bid functions tend to be defined over a series of steps. It would be possible to accommodate a generalization to include piecewise differentiable cost functions with different bid steps, at only the cost of a little notational complexity. None of the important features of the coordination process would be affected by this extension.

With these definitions we can construct the aggregate benefit function in terms of the net loads ($y$) as in:

$$\begin{aligned} B(y)=\underset{d,g}{Max}\; & B^{*}(d)-C(g) \\ & \textit{subject to} \\ & d-g=y. \end{aligned} \tag{13}$$

In short, the aggregate benefit function for net loads is just the value of a simplified economic dispatch between generation and load, with the value of the load and generation determined by the bids of the market participants. If we let $p$ define the constraint multipliers for the balancing constraints in (13), then with differentiability the usual interpretation applies to the corresponding first-order conditions that:

$$\begin{aligned} p &= \nabla B = \nabla B^{*} = p(d)\,, \\ p &= \nabla C = mc(g)\,. \end{aligned} \tag{14}$$

Hence, we have the market clearing price equal to marginal cost of generation at the equilibrium level that balances supply and demand at the net load for each bus.

Given this information in the region, the local system operator can define adjustment bids that provide candidate schedule changes for redispatch that would produce congestion relief. There is a great deal of flexibility in these adjustment bids. The principal requirement would be that the adjustments are defined relative to the current schedules and regional estimate of prices. In other words, given the tentative schedules, $y$, we define

$$A(x) \approx B(y + x) - B(y), \text{ with } \nabla A = \nabla B = p.$$

For example, the regional system operator could identify those locations with incremental and decremental bids relative to the current schedule. The increments and decrements would define relative supply and demand curves as we redispatch, and *A* could be constructed for these adjustments as a benefit function of the same form as in (13).

In the illustrative calculation, there is a simple parameter ε that defines the slope of the increments and decrements relative to the price, *p*. Hence, the adjustment function would be

$$A(x)=(p-\varepsilon x)x.$$

With the introduction of the congestion costs, we can think of the combined effect if the adjustment bids and the external bid as of the form

$$A(x)=(p-\varpi-\varepsilon x)x.$$

In other words, the congestion cost estimates simply shift the apparent net load increment and decrement curve by the amount of the estimated external congestion cost. A similar interpretation applies to the benefit function, *B*, based on market participant bids within the region.

## MODELS FOR APPROXIMATE IMPLEMENTATIONS

Implementation of such a regional coordination process would necessitate using the existing network models and dispatch tools available to the system operators. In addition, there would be a period of transition that would likely result in only partial participation in the coordinated congestion relief protocol. Although such implementation issues will influence the overall effectiveness of the coordinated result, it would be possible to accommodate these matters in a process that would likely improve coordination immediately and allow for expansion of participation without any need for fundamental redesign of the mechanism.

### Network Modeling

The characterization of the constraints in $K_i$ necessarily involves some model of the entire grid, both internal to the region and external. It is well known that this is a difficult problem that often involves a degree of approximation.[25] Although this is not an insignificant matter, the problems are neither created by nor restricted to the application of a market-oriented coordination framework. It will be necessary to deal with the network approximations in any event.

The early TLR methods for calculating PTDF information confront the same issues of network modeling. The NERC use of a limited set of "flowgates" and static

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[25] Shanyou Hao and Alex Papalexopoulos, "External Network Modeling for Optimal Power Flow Applications," IEEE Transactions on Power Systems, Vol. 20, No. 2, May 1995, pp. 825-837.

PTDFs is a compromise that provides a starting point for improved methods of describing the network constraints.

The use of linear approximations for all the constraints and a formulation in terms of real power only would be a natural simplification. It would not be necessary to go all the way to the simplistic DC-load network formulation, which is handy for illustration but too gross an approximation for the real system. However, there is a great deal of experience in dealing with the linearizations of the grid constraints around a given load flow. This experience includes the associated construction of interface limits and other constraints to address the reactive power details that get dropped from the model. Tools like the GE MAPS model have been widely used for many years with network formulations including several thousand buses and many hundreds of contingency constraints. Conceptually, it would be a straightforward matter to adapt tools like this to the market coordination problem.

The most common form of approximation would be in aggregation of buses and lines outside the region, referred to as constructing an "equivalent" but simpler representation of the network. There is a great deal of experience with this practice, and it is well known that the approximation is imperfect. In effect, this requires the regional system operator to set conservative limits in its constraints to protect against the errors of approximation. While this is an important topic with many opportunities for improving the approximations, there is a simple and clear implication for coordination of congestion relief.

We can represent this aggregation and approximation in terms of its affect on the constraint representation. In effect, there is some aggregation function "*a*" that converts the net loads at all buses into a description of the net loads in a smaller "equivalenced" network. In other words, we let

$$\begin{aligned}&(y_1',\cdots,y_n') = a(y_1,\cdots,y_n)\text{ , and}\\&K_i(y_1,\cdots,y_n) \approx K_i'(y_1',\cdots,y_n') \le 0, \quad i = 1,2,\cdots,n.\end{aligned} \tag{15}$$

In this formulation, the variables $y'$ provide the aggregate estimates for net loads on a reduced set of buses and $K'$ contains the network constraints as described for the reduced grid. The limits in the actual regional dispatch, therefore, are set conservatively to insure that the approximate representation produces a dispatch that is within the security constraints. Each region is using its own network model and particular aggregation based on $K'(y')$. However, the coordination process needs the congestion information in terms of $y$. This amounts to requiring the region to disaggregate the schedule adjustments and congestion cost estimates to conform to the common level of aggregation used in the grid. Hence, the regional coordination problem in (11) would use $K'(y')$ and report the congestion prices as for the full grid in $\omega_{ki} = \lambda_k \nabla K_{ki} = \lambda_k \nabla K'_{ki} \nabla a$. Likewise, the aggregate locational adjustments would be restated for the actual locations.

To illustrate, suppose that the simple aggregation for the buses did no more than add up the net loads for a closely related set of buses. Then the congestion cost for the aggregate bus in the equivalenced model would be the congestion cost reported for each of the buses in the set. To the extent that the aggregation affected the parameters of the equivalenced network, so that the function $a(y)$ involved more than just adding up, these marginal effects of the approximation would be included in the translation of the congestion cost estimates. The compromise in terms of the level of aggregation would be based on the extent to which the approximate constraints captured the effect of the security limitations.

Finally, in a very real sense the approximations are the reality. If the regions are interconnected, there is no escape from the necessity to estimate the effects of external actions on each region and constraint. The more approximate the estimate, the more conservative must be the constraint limit needed in order to preserve system reliability. The security coordinators must do something, including making some approximation. Whatever approximations are used, and whatever improvements are made, would be available just as well for the market-based coordination process as they would for a system of administrative curtailments. Gradual improvements could be included as available, without disrupting the market coordination framework.

**Incompatible Systems and Transitions**

The theory of decomposition and regional coordination assumes that each regional system operator is solving its version of (11). In this case, the coordinated regional market solutions would provide a solution for the market equilibrium in the full grid. It would be desirable to have each system operator working within the same framework. However, it would be likely that at least during a period of transition, one or more regions would not be following the same economic dispatch approach. The question of the robustness of the market coordination scheme then arises when one or more system operators follows a different set of rules.

There are many possibilities and a number of questions. Could a system operator choose not to participate at all and not monitor or mange its constraints? Given the interdependence of the grid, this would seem to imply that other system operators would have to take over responsibility for the constraints. This could be accomplished de facto by using an approximate and conservative representation of the constraints. This would limit capacity and trade compared to what would be possible if the presumably better informed system operator in the dissenting region cooperated in the market coordination.

Could an system operator continue to rely on administrative curtailment rules and ignore the prices used for market coordination? This would seem easy enough to for the regional system operator that prefers administrative rules. The only information needed from other regions would be the net loads in $y$ as reported for the initial TLR system. The more serious difficulty would be in the absence of the regional information produced for the benefit of the market coordination process. In particular, the dissenting region's prices and congestion costs would not be available. These would have to be estimated, somehow.

It would be possible for the market coordination process to operate in some regions, at least as long as the other regions provided information about their binding constraints. To illustrate, suppose that there is one external dissenting region with administrative curtailment rules. Call the region "EXT." Assume that all other regions are participating in the market coordination and settlements process. Suppose that after its application of administrative priorities for binding constraints, region EXT has a set of net loads across the grid that it agrees would be feasible, $y^{EXT}$. In addition, suppose region EXT reports the distribution factors for its binding constraints, $\nabla K_{EXT}$. This would not be much more information than the initial NERC TLR required. For convenience, we pick one of the coordinating regions, and give it responsibility for serving as region EXT's proxy. We assume that at least some of the market participants can offer to buy or sell generation in region EXT that would be used in the market coordination process. To distinguish these bids, we identify the corresponding net benefit function as $B_{EXT}$. Since the regional system operator is not participating in the market coordination process, the bids from region EXT might be quite restricted, including the possibility of no market adjustments in region EXT.

With this information, we reformulate the selected coordinating region's problem in (11) as:

$$\begin{aligned}
&\underset{x_1^j,\cdots,x_1^j}{Max} \quad B_j(y_j + x_j^j) + B_{EXT}(y_{EXT} + x_{EXT}^j) + \sum_{i \neq j, EXT} A_i(x_i^j) - \sum_i \varpi_{ji} x_i^j \\
&\textit{subject to} \\
&L(y_1 + x_1^j, \cdots, y_n + x_n^j) = 0, \\
&K_j(y_1 + x_1^j, \cdots, y_n + x_n^j) \leq 0, \\
&(\nabla K_{EXT})(y_1 + x_1^j - y_1^{EXT}, \cdots, y_n + x_n^j - y_n^{EXT}) \leq 0.
\end{aligned} \tag{16}$$

The final linearized constraint for region EXT, monitored by region j, is a standard technique for approximating a constraint relative to a known load flow. This constraint will help ensure that the adjustments of the market coordination process would not later violate a limit that had been met through administrative curtailments directed by a region not participating in the process.[26] The selected coordinating region with responsibility for including the linearized constraint would report adjustment bids and congestion costs that capture its own conditions and those for region EXT, incorporating the effect of the approximate estimate of region EXT's binding constraint. Note that only the selected region *j* would include region EXT's approximate constraint. From the perspective of the remaining regions, EXT would be seen as part of region *j*, and the effect of the constraints "outside" would be captured through the coordinating process, just as before. And to the extent that there is more than one external region, the same rules could apply given the identification of which coordinating region that would be responsible for each external region.

Could a region refuse to participate in the settlements system? The money aversion built into the initial TLR process makes this an important question. At present,

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[26] If the linear approximation is inaccurate, or new constraints arise in region A, it would update its administrative curtailments and estimates of the distribution factors for binding constraints.

there are no prices and no congestion cost settlement system across regions. In the worst case, like now, the problem could create an aggregate revenue deficiency, probably for congestion payments. It would be an easy matter to make sure that generation was purchased and paid for through the simple expedient of casting at least the net flows in and out of a region as bilateral transactions. The harder task would be to make sure that these transactions paid the associated congestion cost implicit in the difference between the locational prices. If people do not have to pay, the prices would provide no market incentive in support of the coordinated congestion relief process.

To go further with these questions, it would appear necessary to get more specific about what the regional system operators can and cannot do, in order to design approximations that would capture some of the benefits of a market-oriented coordination system. However, it should be possible to make a gradual transition to an improved coordination and congestion relief process, starting with a few cooperating regions. The first steps should make things better, and the approach would proceed, region by region, without requiring a big-bang solution. However, the difficulties could be significant. It would be better if all were working from the same page.

### Settlements and Revenue Adequacy

The use of pricing for congestion relief suggests the use of financial transmission rights to hedge the price differential between location. This raises the question of the revenue adequacy of the congestion payments mechanism. Naturally, the result depends on the nature of the settlements system.

The easiest case conceptually is when there is a coordinated allocation of financial transmission rights that is simultaneously feasible with respect to all the constraints. This converts the apparent regional model into a single virtual region, and the usual results would apply. The aggregate payments under the settlement system would be revenue adequate, and nothing fundamentally new would be introduced. [27]

Here we focus on the case of congestion only and define a candidate settlement system that would provide revenue adequacy in the case that each system operator was offering its own set of transmission congestion contracts. The transmission congestion contracts would be feasible with respect to its own constraints, but the regional system operator says nothing about the feasibility elsewhere. In addition, the regional transmission congestion contract provides a right to collect only the congestion costs that arise from the constraints in the region.

A natural settlement system would be for the system operators to pay into the settlement system the net collections and then for each region k to receive payments equal to the congestion costs associated with the constraints from its region.

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[27] Scott M. Harvey, William W. Hogan, and Susan L. Pope, "Transmission Capacity Reservations and Transmission Congestion Contracts," Center for Business and Government, Harvard University, June 6, 1996, (Revised March 8, 1997).

$$\omega_k y = \sum_i \omega_{ki} y_i \ . \tag{17}$$

These congestion costs are determined from (11), which is an economic dispatch problem. By assumption, the transmission congestion contracts would meet the constraints of this problem. Hence, the usual argument would apply to show that the congestion revenues would be sufficient to make the payments under the transmission congestion contracts.

In other words, if $TCC_k$ is the aggregate set of feasible transmission congestion contracts applicable for region k, then the system operator has revenues $\omega_k y$ and obligations $\omega_k TCC_k$. The basic revenue adequacy results is that $\omega_k y \geq \omega_k TCC_k$.

The same settlements system could apply to either form of transmission congestion contract, with each regional system operator effectively making payments under the transmission congestion contracts for its region, keeping any surplus congestion rents arising from its constraints as now to reduce access charges or share among those paying for the transmission grid. Of course, the settlements system could be handled in such a way that all this detail is invisible to the market participants, who receive payments only for the net effect on congestion.

## Example Iterations

The following table summarizes key variables for the example application of the three region coordination on locational prices. The constraint multipliers are for either the upper or lower bound, whichever is binding.

**Multi-Regional Coordination of Constrained Equilibrium**

| | | Start | Iteration 1 | ------------ | Iteration 1 | Iteration 2 | ------------ | Iteration 2 | Iteration 3 | ------------ | Iteration 3 | Final |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Region | | | I | II | III | I | II | III | I | II | III | |
| Bus | | | | | | | | | | | | |
| 1 | p | 50.00 | 83.00 | 82.37 | 82.30 | 83.00 | 82.94 | 82.95 | 83.00 | 83.00 | 83.00 | 83.00 |
| | q | 2000 | 900 | 921 | 923 | 900 | 902 | 902 | 900 | 900 | 900 | 900 |
| 2 | p | 50.00 | 33.78 | 33.38 | 33.62 | 32.66 | 32.60 | 32.63 | 32.90 | 32.90 | 32.90 | 32.90 |
| | q | -1000 | -459 | -446 | -454 | -422 | -420 | -421 | -430 | -430 | -430 | -430 |
| 3 | p | 50.00 | 43.46 | 42.63 | 42.24 | 42.82 | 42.75 | 42.75 | 42.38 | 42.38 | 42.38 | 42.38 |
| | q | -1000 | -782 | -754 | -741 | -761 | -758 | -758 | -746 | -746 | -746 | -746 |
| 4 | p | 50.00 | 48.68 | 31.50 | 31.43 | 31.60 | 31.90 | 31.95 | 31.95 | 32.00 | 32.00 | 32.00 |
| | q | -1000 | -956 | -383 | -381 | -387 | -397 | -398 | -398 | -400 | -400 | -400 |
| 5 | p | 50.00 | 48.57 | 71.54 | 71.32 | 71.32 | 72.39 | 72.43 | 72.44 | 72.38 | 72.38 | 72.38 |
| | q | 2000 | 2048 | 1282 | 1289 | 1289 | 1254 | 1252 | 1252 | 1254 | 1254 | 1254 |
| 6 | p | 50.00 | 48.46 | 48.28 | 47.93 | 47.77 | 46.93 | 46.97 | 46.98 | 46.99 | 47.00 | 47.00 |
| | q | -1000 | -949 | -943 | -931 | -926 | -898 | -899 | -899 | -900 | -900 | -900 |
| 7 | p | 50.00 | 47.90 | 46.35 | 44.03 | 43.86 | 43.69 | 43.27 | 43.28 | 43.29 | 43.29 | 43.30 |
| | q | -1000 | -930 | -878 | -801 | -795 | -790 | -776 | -776 | -776 | -776 | -777 |
| 8 | p | 50.00 | 48.13 | 47.19 | 36.91 | 36.86 | 36.81 | 36.64 | 36.63 | 36.63 | 36.64 | 36.65 |
| | q | -1000 | -938 | -906 | -564 | -562 | -560 | -555 | -554 | -554 | -555 | -555 |
| 9 | p | 50.00 | 48.01 | 46.77 | 60.23 | 60.12 | 60.01 | 60.43 | 60.43 | 60.43 | 60.40 | 60.40 |
| | q | 2000 | 2066 | 2108 | 1659 | 1663 | 1666 | 1652 | 1652 | 1652 | 1653 | 1653 |
| Line | $\lambda_1$ | 0.00 | 61.12 | 61.12 | 61.12 | 70.76 | 70.76 | 70.76 | 70.00 | 70.00 | 70.00 | 69.98 |
| | $\lambda_2$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | $\lambda_3$ | 0.00 | 27.64 | 27.64 | 27.64 | 19.77 | 19.77 | 19.77 | 20.75 | 20.75 | 20.75 | 20.79 |
| | $\lambda_4$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | $\lambda_5$ | 0.00 | 0.00 | 63.26 | 63.26 | 63.26 | 65.95 | 65.95 | 65.95 | 65.76 | 65.76 | 65.73 |
| | $\lambda_6$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | $\lambda_7$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | $\lambda_8$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | $\lambda_9$ | 0.00 | 0.00 | 0.00 | 39.52 | 39.52 | 39.52 | 40.95 | 40.95 | 40.95 | 40.81 | 40.78 |
| | $\lambda_{10}$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | $\lambda_{11}$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | $\lambda_{12}$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

The following table reports the distribution factors. For each element, this is the impact on the flow on the corresponding line induced by an increase in net load at the bus balanced by a reduction in the net load at bus 1, which is the arbitrary reference bus for the transmission constraints.

**Distribution Factors for Buses Relative to Bus 1**

| Bus | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| Line | | | | | | | | | |
| 1 to 2 | 0 | 0.644 | 0.356 | 0.422 | 0.444 | 0.467 | 0.578 | 0.533 | 0.556 |
| 2 to 3 | 0 | -0.289 | 0.289 | 0.156 | 0.111 | 0.067 | -0.156 | -0.067 | -0.111 |
| 1 to 3 | 0 | 0.356 | 0.644 | 0.578 | 0.556 | 0.533 | 0.422 | 0.467 | 0.444 |
| 3 to 4 | 0 | 0.067 | -0.067 | 0.733 | 0.667 | 0.600 | 0.267 | 0.400 | 0.333 |
| 4 to 5 | 0 | 0.022 | -0.022 | -0.089 | 0.556 | 0.200 | 0.089 | 0.133 | 0.111 |
| 4 to 6 | 0 | 0.044 | -0.044 | -0.178 | 0.111 | 0.400 | 0.178 | 0.267 | 0.222 |
| 5 to 6 | 0 | 0.022 | -0.022 | -0.089 | -0.444 | 0.200 | 0.089 | 0.133 | 0.111 |
| 6 to 8 | 0 | 0.067 | -0.067 | -0.267 | -0.333 | -0.400 | 0.267 | 0.400 | 0.333 |
| 8 to 9 | 0 | 0.022 | -0.022 | -0.089 | -0.111 | -0.133 | 0.089 | -0.200 | 0.444 |
| 7 to 8 | 0 | -0.044 | 0.044 | 0.178 | 0.222 | 0.267 | -0.178 | 0.400 | 0.111 |
| 7 to 9 | 0 | -0.022 | 0.022 | 0.089 | 0.111 | 0.133 | -0.089 | 0.200 | 0.556 |
| 2 to 7 | 0 | -0.067 | 0.067 | 0.267 | 0.333 | 0.400 | 0.733 | 0.600 | 0.667 |

The shift factors describe the impacts on lines flows. Hence, these apply to the upper bound constraints in the direction indicted for each line, $\nabla K$. The corresponding shift factors for the lower bound constraints are the negatives of those in the table.